Algebra and Polynomials

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Algebra and Polynomials

Polynomial algebra underpins most of Section 1. Factorisation and the Factor Theorem appear directly in Paper 01 items and form the setup for quadratic, inequality, and calculus questions on Paper 02.

Polynomial Terminology

A **polynomial** in x is an expression of the form

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where the powers are non-negative integers. The **degree** is the highest power. A polynomial of degree 4 or below is the focus at this level.

Degree	Name	Example
1	Linear	[Math: 3x - 7]
2	Quadratic	[Math: 2x^2 + x - 5]
3	Cubic	[Math: x^3 - 4x^2 + 2]
4	Quartic	[Math: x^4 - x^2 + 1]

The **leading coefficient** is the coefficient of the highest-power term. The **constant term** is the term with no x.

Polynomial Long Division

When dividing a polynomial $p(x)$ by a divisor $d(x)$, the result takes the form:

$$p(x) = d(x) \cdot q(x) + r(x)$$

where $q(x)$ is the **quotient** and $r(x)$ is the **remainder**

. The degree of the remainder is strictly less than the degree of the divisor. For a linear divisor $(x-a)$, the remainder is always a constant.

📋 **Example**

Divide $p(x)=2x^3-3x^2+x+5$ by $(x-2)$.

<LongDivision dividend="2x^3 - 3x^2 + x + 5" divisor="x - 2" />

$$2x^3-3x^2+x+5=(x-2)(2x^2+x+3)+11$$

The Remainder Theorem

Long division works, but it's slow. The Remainder Theorem gives you the remainder in one step.

Why it works: When you divide $p(x)$ by $(x-a)$, the result is always of the form:

$$p(x)=(x-a)\,q(x)+r$$

Now substitute $x=a$ into both sides. The left side gives $p(a)$. The right side gives $(a-a)\,q(a)+r=0+r=r$.

So $p(a)=r$. That is the entire theorem.

🧠 **Remember**

Remainder Theorem: When $p(x)$ is divided by $(x-a)$, the remainder equals $p(a)$.

To find a : set the divisor equal to zero and solve. For $(x+3)$, set $x+3=0$ to get $x=-3$, so evaluate $p(-3)$.

Past Paper Worked Example

📋 **Example**

CSEC 2021, Question 1(a)

$f(x) = ax^3 + 7x^2 - 7x - 3$.

(i) Determine the remainder when $f(x)$ is divided by $(x-1)$.

Set $x - 1 = 0 \Rightarrow x = 1$.

$$f(1) = a(1)^3 + 7(1)^2 - 7(1) - 3 = a + 7 - 7 - 3 = a - 3$$

The remainder when divided by $(x-1)$ is $a - 3$.

(ii) If the remainder when $f(x)$ is divided by $(x+3)$ equals the remainder from part (i), find a.

Set $x + 3 = 0 \Rightarrow x = -3$.

$$f(-3) = a(-27) + 7(9) - 7(-3) - 3 = -27a + 63 + 21 - 3 = -27a + 81$$

Set this equal to the remainder from part (i):

$$-27a + 81 = a - 3$$

$$84 = 28a$$

$$a = 3$$

When Two Divisions Give the Same Remainder

The 2022 paper gave a similar "same remainder" structure, a useful pattern to recognise.

📋 **Example**

CSEC 2022, Question 2(a)

When $p(x) = 2x^3 - 3x^2 - cx + d$ is divided by $(x + 1)$ and $(x - 2)$, the remainder is 64 in both cases. Find c and d.

From $(x + 1)$, substitute $x = -1$:

$$p(-1) = 2(-1) - 3(1) - c(-1) + d = -2 - 3 + c + d = c + d - 5 = 64$$

$$c + d = 69 \quad \cdots (1)$$

From $(x - 2)$, substitute $x = 2$:

$$p(2) = 2(8) - 3(4) - 2c + d = 16 - 12 - 2c + d = 4 - 2c + d = 64$$

$$-2c + d = 60 \quad \cdots (2)$$

Subtract (2) from (1): $3c = 9 \Rightarrow c = 3$. Then $d = 69 - 3 = 66$.

The Factor Theorem

The Factor Theorem follows directly from the Remainder Theorem. If you divide $p(x)$ by $(x - a)$ and the remainder is **zero**, then $(x - a)$ divides evenly, meaning it is a **factor**.

Since the Remainder Theorem tells us the remainder equals $p(a)$, a zero remainder simply means $p(a) = 0$. So:

(x - a) is a factor of p(x) if and only if p(a) = 0.

This works in both directions:

- To **check** whether $(x - a)$ is a factor, evaluate $p(a)$. If the result is zero, it is a factor.
- To **find** factors, try values of a until $p(a) = 0$.

📋 **Example**

Show that $(x-3)$ is a factor of $p(x)=x^3-x^2-5x-3$, then fully factorise $p(x)$.

Step 1: Confirm the factor:

$p(3)=27-9-15-3=0$. Since $p(3)=0$, $(x-3)$ is a factor.

Step 2: Divide to find the remaining factor:

<LongDivision dividend="x^3 - x^2 - 5x - 3" divisor="x - 3" />

Step 3: Factorise the quadratic:

$x^2+2x+1=(x+1)^2$

$$p(x)=(x-3)(x+1)^2$$

Past Paper: Full Factorisation from a Given Factor

📋 **Example**

CSEC 2024, Question 1(a)(i)

Determine all linear factors of $p(x)=3x^3+8x^2-20x-16$, given that $(x-2)$ is a factor.

Step 1: Divide by the known factor $(x-2)$:

<LongDivision dividend="3x^3 + 8x^2 - 20x - 16" divisor="x - 2" />

Step 2: Factorise the quadratic $3x^2+14x+8$:

Find two numbers with product $3\times 8=24$ and sum 14: these are 2 and 12.

$$3x^2+14x+8=3x^2+2x+12x+8=x(3x+2)+4(3x+2)=(3x+2)(x+4)$$

Result:

$$p(x)=(x-2)(3x+2)(x+4)$$

Finding Unknown Coefficients

If $(x-a)$ is a known factor of $p(x)$, then $p(a)=0$

gives an equation you can solve for any unknown coefficient. If the problem gives two conditions (two known factors, or a factor and a known remainder), apply the theorem twice and solve the resulting simultaneous equations.

Trial is only needed when the question asks you to fully factorise a polynomial and gives you no starting factor. If a factor is stated, or the question only asks for a remainder, you can go straight to the relevant theorem. Trial comes up on Paper 02 questions phrased as "factorise completely" or "find all linear factors."

> 📝 **Exam Tip**
>
> When you need to find a factor by trial, you are literally testing whether $(x-1)$, $(x+1)$, $(x-2)$, $(x+2)$, and so on are factors. You do this by substituting $x=1$, $x=-1$, $x=2$, $x=-2$ into $p(x)$ and checking if the result is zero.
>
> For a polynomial like $p(x)=2x^3-3x^2-11x+6$, the values worth trying are $\pm1, \pm2, \pm3, \pm6$ (factors of the constant term 6) and $\pm\frac{1}{2}, \pm\frac{3}{2}$ (those divided by the leading coefficient 2). Start with $x=1$ and $x=-1$ since they are the quickest to calculate.