Coordinate Geometry

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Coordinate Geometry

Coordinate geometry translates geometric problems into algebra that can be solved with equations. At this level it covers two main areas: **straight lines** (extending CSEC Maths) and **circles** (new at Additional Mathematics level).

Key Formulas

For two points $A(x1, y1)$ and $B(x2, y2)$:

Quantity	Formula
Gradient	[Math: m = \dfrac{y2 - y1}{x2 - x1}]
Midpoint	[Math: M = \left(\dfrac{x1+x2}{2},\, \dfrac{y1+y2}{2}\right)]
Distance	[Math: \lvert AB\rvert = \sqrt{(x2-x1)^2 + (y2-y1)^2}]

The distance formula is Pythagoras applied to the horizontal and vertical separations.

Equations of Straight Lines

A straight line can be written in several equivalent forms:

- **Slope-intercept:** $y = mx + c$ (gradient m , y -intercept c)
- **Point-slope:** $y - y1 = m(x - x1)$ (given gradient m and one point)
- **General:** $ax + by + c = 0$

Finding the equation given two points: calculate the gradient first, then substitute one point into point-slope form.

📋 **Example**

Find the equation of the line through $(1, -3)$ and $(4, 6)$.

$$m = \frac{6 - (-3)}{4 - 1} = \frac{9}{3} = 3$$

Using point-slope with $(1, -3)$: $y + 3 = 3(x - 1) \Rightarrow y = 3x - 6$

Parallel and Perpendicular Lines

- **Parallel lines** have **equal** gradients: $m1 = m2$.
- **Perpendicular lines** have gradients whose product is -1 : $m1m2 = -1$

, equivalently $m2 = -1/m1$.

📋 **Example**

Find the equation of the line perpendicular to $y = \frac{2}{3}x + 5$ passing through $(4, 1)$.

The given gradient is $\frac{2}{3}$. The perpendicular gradient is $-\frac{3}{2}$.

$$y - 1 = -\frac{3}{2}(x - 4) \Rightarrow y = -\frac{3}{2}x + 7$$

Perpendicular Bisector

The perpendicular bisector of a line segment AB passes through the midpoint of AB and is perpendicular to AB.

📋 **Example**

Find the equation of the perpendicular bisector of the segment joining $(2, 1)$ and $(8, 5)$.

Midpoint: $(5, 3)$. Gradient of segment: $\frac{5-1}{8-2} = \frac{2}{3}$.

Perpendicular gradient: $-\frac{3}{2}$.

$$y - 3 = -\frac{3}{2}(x - 5) \Rightarrow y = -\frac{3}{2}x + \frac{21}{2}$$

Intersection of Two Straight Lines

To find the point where two lines meet, solve their equations simultaneously — set the y expressions for
equal and solve for x, then back-substitute.

> 📋 **Example**
> Find the point of intersection of $y = 2x - 1$ and $y = -x + 5$.
>
> Setting equal: $2x - 1 = -x + 5 \Rightarrow 3x = 6 \Rightarrow x = 2$
>
> $y = 2(2) - 1 = 3$. Intersection: $(2, 3)$.

Circles

Standard Form

The equation of a circle with centre (a, b) and radius r:

$$(x-a)^2 + (y-b)^2 = r^2$$

Every point on the circle is exactly r
units from the centre, so this equation comes directly from the distance formula.

> 📝 **Exam Tip**
> The signs inside the brackets are subtracted. $(x-3)^2 + (y+2)^2 = 25$ has centre $(3, -2)$, not $(-3, 2)$. Read the sign that makes each bracket zero.

General Form

Expanding the standard form gives:

$$x^2 + y^2 + 2fx + 2gy + c = 0$$

In this form the centre is $(-f, -g)$ and the radius is $\sqrt{f^2 + g^2 - c}$
, but you do not need to memorise these formulas. It is safer and more reliable to x y

complete the squarefor both and

, which converts general form back to standard form directly.

📋 **Example**

Find the centre and radius of $x^2 + y^2 - 6x + 4y - 3 = 0$.

Complete the square:

$$(x^2 - 6x + 9) + (y^2 + 4y + 4) = 3 + 9 + 4$$

$$(x - 3)^2 + (y + 2)^2 = 16$$

Centre: $(3, -2)$. Radius: 4.

🧠 **Remember**

When completing the square to find a circle's radius, the right-hand side must be strictly positive. If it is zero, the "circle" is a single point. If it is negative, no real circle exists.

Tangents and Normals to Circles

A **tangent**to a circle at a point P is perpendicular to the radius at P.

Method for finding the tangent at P:

- 1. Find the gradient of the radius from the centre to P.
- 2. The tangent gradient is the negative reciprocal.
- 3. Write the tangent equation using point-slope form with point P.

📋 **Example**

Find the equation of the tangent to the circle $(x - 1)^2 + (y + 2)^2 = 25$ at the point $(4, 2)$.

Centre: $(1, -2)$. Gradient of radius to $(4, 2)$: $\frac{2-(-2)}{4-1} = \frac{4}{3}$.

Tangent gradient: $-\frac{3}{4}$.

$$y - 2 = -\frac{3}{4}(x - 4) \Rightarrow y = -\frac{3}{4}x + 5$$

The **normal**at P

passes through the centre. Its equation uses the radius gradient (not the perpendicular gradient).

Intersection of a Line and a Circle

Substitute the line's equation into the circle's equation to get a quadratic in one variable. The discriminant of that quadratic tells you about the intersection:

- $\Delta > 0$: two intersection points (line is a secant).
- $\Delta = 0$: one intersection point (line is a tangent).
- $\Delta < 0$: no intersection.

> 📋 **Example**
> Find where the line $y = x + 1$ meets the circle $x^2 + y^2 = 13$.
>
> Substitute $y = x + 1$:
>
> $$x^2 + (x+1)^2 = 13 \Rightarrow 2x^2 + 2x - 12 = 0 \Rightarrow x^2 + x - 6 = 0 \Rightarrow (x+3)(x-2) = 0$$
>
> $x = -3, y = -2$ and $x = 2, y = 3$.

Collinearity

Three points A, B, C are **collinear** if the gradient of AB equals the gradient of BC (or equivalently, the gradient of AC). Alternatively, show that $\overrightarrow{AB}$ is a scalar multiple of $\overrightarrow{AC}$.