

Additional Mathematics Formula Book

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Section 1: Algebra, Sequences and Series

A. Algebra

Remainder Theorem: When $f(x)$ is divided by $(x - a)$, the remainder is $f(a)$.

Factor Theorem: $(x - a)$ is a factor of $f(x)$ if and only if $f(a) = 0$.

B. Quadratics

For $ax^2 + bx + c = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Vertex form $a(x + h)^2 + k$, vertex at $(-h, k)$:

$$h = \frac{b}{2a} \quad k = c - ah^2$$

Discriminant $\Delta = b^2 - 4ac$:

[Math: \Delta]	Nature of roots
[Math: \Delta > 0]	Two distinct real roots
[Math: \Delta = 0]	One repeated real root
[Math: \Delta < 0]	No real roots

Vieta's formulas for roots α and β of $ax^2 + bx + c = 0$:

$$\alpha + \beta = -\frac{b}{a} \quad \alpha\beta = \frac{c}{a}$$

Expression	How to evaluate
$[\text{Math: } \alpha^2 + \beta^2]$	$[\text{Math: } (\alpha + \beta)^2 - 2\alpha\beta]$
$[\text{Math: } (\alpha - \beta)^2]$	$[\text{Math: } (\alpha + \beta)^2 - 4\alpha\beta]$
$[\text{Math: } \frac{1}{\alpha} + \frac{1}{\beta}]$	$[\text{Math: } \frac{\alpha + \beta}{\alpha\beta}]$
$[\text{Math: } \alpha^2\beta + \alpha\beta^2]$	$[\text{Math: } \alpha\beta(\alpha + \beta)]$

Forming a new quadratic from known sum S and product P :

$$x^2 - Sx + P = 0$$

Multiply through by a constant if needed so that $a, b, c \in \mathbb{Z}$.

C. Inequalities

Quadratic inequalities: find the critical values (roots), sketch the parabola, read the solution set from the graph.

Rational inequalities $\frac{f(x)}{g(x)} \leq k$

: rearrange to get zero on one side, find critical values (roots and excluded values), apply a sign diagram.

D. Surds, Indices, and Logarithms

Surd rules:

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab} \quad \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}} \quad \sqrt{a} \times \sqrt{a} = a$$

Rationalise using the conjugate: $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$.

Laws of indices:

Law	Rule

$[Math: a^m \times a^n]$	$[Math: a^{m+n}]$
$[Math: a^m \div a^n]$	$[Math: a^{m-n}]$
$[Math: (a^m)^n]$	$[Math: a^{mn}]$
$[Math: a^0]$	$[Math: 1]$
$[Math: a^{-n}]$	$[Math: \frac{1}{a^n}]$
$[Math: a^{m/n}]$	$[Math: \sqrt[n]{a^m}]$
$[Math: (ab)^m]$	$[Math: a^m b^m]$

If $a^m = a^n$ and $a \neq 0, 1$, then $m = n$.

Logarithm definition: $a^x = b \iff \log_a b = x$ ($a > 0, a \neq 1, b > 0$)

Key values: $\log_a 1 = 0, \log_a a = 1$.

Laws of logarithms:

Law	Statement
Product	$[Math: \log_a(xy) = \log_a x + \log_a y]$
Quotient	$[Math: \log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y]$
Power	$[Math: \log_a(x^n) = n \log_a x]$

To solve $a^x = b$: apply **log** to both sides and use the power law: $x = \frac{\log b}{\log a}$.

Linearisation:

Relationship	Linearised form	Plot	Gradient	Intercept
$[Math: y = ab^x]$	$[Math: \log y = x \log b + \log a]$	$[Math: \log y]$ vs $[Math: x]$	$[Math: \log b]$	$[Math: \log a]$
$[Math: y = ax^n]$	$[Math: \log y = n \log x + \log a]$	$[Math: \log y]$ vs $[Math: \log x]$	$[Math: n]$	$[Math: \log a]$

E. Sequences and Series

Sigma notation: $\sum_{r=1}^n U_r$ means the sum of U_r from $r = 1$ to $r = n$.

Arithmetic sequences (a = first term, d = common difference, l = last term, n terms):

$$T_n = a + (n - 1)d \quad S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

Geometric sequences (a = first term, r = common ratio):

$$T_n = ar^{n-1} \quad r = \frac{T_{n+1}}{T_n} \quad S_n = \frac{a(1 - r^n)}{1 - r}$$

Sum to infinity(converges when $|r| < 1$ only):

$$S_\infty = \frac{a}{1 - r}$$

Section 2: Coordinate Geometry, Vectors, and Trigonometry

A. Coordinate Geometry

For two points (x_1, y_1) and (x_2, y_2) :

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Parallel lines: equal gradients. Perpendicular lines: $m_1 m_2 = -1$.

Equation of a line: $y - y_1 = m(x - x_1)$

Circle equations:

Form	Equation	Centre	Radius
Standard	$(x - a)^2 + (y - b)^2 = r^2$	(a, b)	r
General	$x^2 + y^2 + 2fx + 2gy + c = 0$	$(-f, -g)$	$\sqrt{f^2 + g^2 - c}$

The tangent at a point on a circle is perpendicular to the radius at that point.

B. Vectors

For $\mathbf{v} = \begin{pmatrix} x \\ y \end{pmatrix}$:

$$|\mathbf{v}| = \sqrt{x^2 + y^2} \quad \hat{\mathbf{v}} = \frac{\mathbf{v}}{|\mathbf{v}|}$$

Dot product:

$$\begin{pmatrix} a \\ b \end{pmatrix} \cdot \begin{pmatrix} c \\ d \end{pmatrix} = ac + bd$$

Angle between vectors:

$$\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$$

- $\mathbf{a} \cdot \mathbf{b} = 0$: vectors are perpendicular.
- $\mathbf{b} = k\mathbf{a}$ for some scalar k : vectors are parallel.
- Three points A , B , C are collinear if $\overrightarrow{AB} = k\overrightarrow{AC}$.

Direction of a vector: $\theta = \arctan\left(\frac{y}{x}\right)$, then adjust for the correct quadrant (add 180° in Q2 or Q3).

C. Trigonometry

Conversion:

$$\theta^\circ \times \frac{\pi}{180} \rightarrow \text{rad} \quad \theta \text{ rad} \times \frac{180}{\pi} \rightarrow ^\circ$$

Arc and sector (θ in radians):

$$l = r\theta \quad A_{\text{sector}} = \frac{1}{2}r^2\theta \quad A_{\text{segment}} = \frac{1}{2}r^2(\theta - \sin \theta)$$

Exact values:

θ	0	30°	45°	60°	90°
$\sin \theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undef.

CAST rule: All (Q1), Sine (Q2), Tangent (Q3), Cosine (Q4) are positive.

Graph properties:

Function	Period	Amplitude
$y = a \sin(kx)$	$\frac{2\pi}{k}$	$ a $
$y = a \cos(kx)$	$\frac{2\pi}{k}$	$ a $
$y = a \tan(kx)$	$\frac{\pi}{k}$	—

Pythagorean identities:

$$\sin^2 \theta + \cos^2 \theta = 1 \quad \tan^2 \theta + 1 = \sec^2 \theta$$

Compound angle formulas:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Double angle formulas:

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Section 3: Introductory Calculus

A. Differentiation

[Math: f(x)]	[Math: f'(x)]
[Math: x^n]	[Math: nx^{n-1}]
[Math: \sin(ax)]	[Math: a\cos(ax)]
[Math: \cos(ax)]	[Math: -a\sin(ax)]

Chain rule(let $u = g(x)$, $y = f(u)$):

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Product rule($y = uv$):

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

Quotient rule($y = \frac{u}{v}$):

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Stationary points:set $f'(x) = 0$, solve for x , find y .

Second derivative at stationary point	Nature
[Math: f''(x) > 0]	Minimum

[Math: $f''(x) < 0$]	Maximum
[Math: $f''(x) = 0$]	Inconclusive; use sign-change test on [Math: $f'(x)$]

Tangent and normal at (x_0, y_0) : tangent gradient $m_T = f'(x_0)$; normal gradient

$$m_N = -\frac{1}{m_T}$$

Connected rates: $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$

B. Integration

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad (n \neq -1)$$

$$\int (ax + b)^n dx = \frac{(ax + b)^{n+1}}{a(n+1)} + c$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c \quad \int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

Area under a curve between $x = a$ and $x = b$:

$$A = \int_a^b y dx$$

Area between two curves ($f(x) \geq g(x)$ on $[a, b]$):

$$A = \int_a^b [f(x) - g(x)] dx$$

Volume of revolution about the x -axis:

$$V = \pi \int_a^b y^2 dx$$

Kinematics

Displacement s , velocity v , acceleration a , time t :

$$v = \frac{ds}{dt} \quad a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

$$s = \int v dt \quad v = \int a dt$$

SUVAT equations(uniform acceleration only; u = initial velocity, s = displacement):

$$v = u + at \quad s = ut + \frac{1}{2}at^2 \quad v^2 = u^2 + 2as \quad s = \frac{1}{2}(u + v)t$$

Particle is stationary when $v = 0$. Direction changes when v changes sign.

Section 4: Probability and Statistics

A. Data Representation and Analysis

Mean:

$$\bar{x} = \frac{\sum x}{n} \quad (\text{ungrouped}) \quad \bar{x} = \frac{\sum fx}{\sum f} \quad (\text{grouped})$$

Variance and standard deviation:

$$S^2 = \frac{\sum (x - \bar{x})^2}{n} \quad S = \sqrt{\frac{\sum (x - \bar{x})^2}{n}} \quad (\text{ungrouped})$$

$$S^2 = \frac{\sum f(x - \bar{x})^2}{\sum f} \quad (\text{grouped})$$

Measures of spread:

$$\text{Range} = x_{\max} - x_{\min} \quad \text{IQR} = Q_3 - Q_1 \quad \text{Semi-IQR} = \frac{Q_3 - Q_1}{2}$$

Skewness:

Box plot pattern	Skew	Order of averages
Right whisker longer / [Math: $Q_3 - Q_2 > Q_2 - Q_1$]	Positive	Mode < Median < Mean
Left whisker longer / [Math: $Q_3 - Q_2 < Q_2 - Q_1$]	Negative	Mean < Median < Mode
Symmetric	None	Mode = Median = Mean

B. Probability Theory

$$P(A) = \frac{\text{favourable outcomes}}{\text{total outcomes}} \quad P(A') = 1 - P(A) \quad 0 \leq P(A) \leq 1$$

Rule	Formula
Addition (general)	[Math: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$]
Mutually exclusive ([Math: $A \cap B = \emptyset$])	[Math: $P(A \cup B) = P(A) + P(B)$]
Conditional	[Math: $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$]
Independent	[Math: $P(A \cap B) = P(A) \cdot P(B)$]

Two non-zero-probability events cannot be simultaneously mutually exclusive and independent.

Tree diagrams: multiply along branches for intersection probabilities; add across branches for union. The syllabus restricts trees to two initial branches.

Venn diagrams: restricted to two sets; the four regions (A only, B only, $A \cap B$, neither) must sum to $P(S) = 1$.