Additional Mathematics Formula Book

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Section 1: Algebra, Sequences and Series

A. Algebra

Remainder Theorem: When $f(x)$ is divided by $(x-a)$, the remainder is $f(a)$.

Factor Theorem: $(x-a)$ is a factor of $f(x)$ if and only if $f(a)=0$.

B. Quadratics

For $ax^2+bx+c=0$:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Vertex form $a(x+h)^2+k$, vertex at $(-h,\,k)$:

$$h=\frac{b}{2a}\qquad k=c-ah^2$$

Discriminant $\Delta=b^2-4ac$:

[Math: \Delta]	Nature of roots
[Math: \Delta > 0]	Two distinct real roots
[Math: \Delta = 0]	One repeated real root
[Math: \Delta < 0]	No real roots

Vieta's formulasfor roots α and β of $ax^2+bx+c=0$:

$$\alpha + \beta = -\frac{b}{a} \qquad \alpha\beta = \frac{c}{a}$$

Expression	How to evaluate
[Math: \alpha^2 + \beta^2]	[Math: (\alpha + \beta)^2 - 2\alpha\beta]
[Math: (\alpha - \beta)^2]	[Math: (\alpha + \beta)^2 - 4\alpha\beta]
[Math: \dfrac{1}{\alpha} + \dfrac{1}{\beta}]	[Math: \dfrac{\alpha + \beta}{\alpha\beta}]
[Math: \alpha^2\beta + \alpha\beta^2]	[Math: \alpha\beta(\alpha + \beta)]

Forming a new quadraticfrom known sum S and product P:

$$x^2 - Sx + P = 0$$

Multiply through by a constant if needed so that $a, b, c \in \mathbb{Z}$.

C. Inequalities

Quadratic inequalities: find the critical values (roots), sketch the parabola, read the solution set from the graph.

Rational inequalities $\frac{f(x)}{g(x)} \lessgtr k$
: rearrange to get zero on one side, find critical values (roots and excluded values), apply a sign diagram.

D. Surds, Indices, and Logarithms

Surd rules:

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab} \qquad \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}} \qquad \sqrt{a} \times \sqrt{a} = a$$

Rationalise using the conjugate: $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$.

Laws of indices:

Law	Rule

[Math: a^m \times a^n]	[Math: a^{m+n}]
[Math: a^m \div a^n]	[Math: a^{m-n}]
[Math: (a^m)^n]	[Math: a^{mn}]
[Math: a^0]	[Math: 1]
[Math: a^{-n}]	[Math: \dfrac{1}{a^n}]
[Math: a^{m/n}]	[Math: \sqrt[n]{a^m}]
[Math: (ab)^m]	[Math: a^m b^m]

If $a^m = a^n$ and $a \neq 0, 1$, then $m = n$.

Logarithm definition: $a^x = b \iff \log_a b = x \quad (a > 0,\ a \neq 1,\ b > 0)$

Key values: $\log_a 1 = 0$, $\log_a a = 1$.

Laws of logarithms:

Law	Statement
Product	[Math: \loga(xy) = \loga x + \log_a y]
Quotient	[Math: \loga\!\left(\dfrac{x}{y}\right) = \loga x - \log_a y]
Power	[Math: \loga(x^n) = n\loga x]

To solve $a^x = b$: apply $\log$ to both sides and use the power law: $x = \frac{\log b}{\log a}$.

Linearisation:

Relationship	Linearised form	Plot	Gradient	Intercept
[Math: y = ab^x]	[Math: \log y = x\log b + \log a]	[Math: \log y] vs [Math: x]	[Math: \log b]	[Math: \log a]
[Math: y = ax^n]	[Math: \log y = n\log x + \log a]	[Math: \log y] vs [Math: \log x]	[Math: n]	[Math: \log a]

E. Sequences and Series

Sigma notation: $\sum r = 1^n Ur$ means the sum of U_r from $r = 1$ to $r = n$.

Arithmetic sequences(a = first term, d = common difference, l = last term, n terms):

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a + l)$$

Geometric sequences(a = first term, r = common ratio):

$$T_n = ar^{n-1} \qquad r = \frac{T_{n+1}}{T_n} \qquad S_n = \frac{a(1 - r^n)}{1 - r}$$

Sum to infinity(converges when $|r| < 1$ only):

$$S_\infty = \frac{a}{1 - r}$$

Section 2: Coordinate Geometry, Vectors, and Trigonometry

A. Coordinate Geometry

For two points $(x1, y1)$ and $(x2, y2)$:

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Parallel lines: equal gradients. Perpendicular lines: $m1m2 = -1$.

Equation of a line: $y - y1 = m(x - x1)$

Circle equations:

Form	Equation	Centre	Radius
Standard	[Math: (x - a)^2 + (y - b)^2 = r^2]	[Math: (a, b)]	[Math: r]
General	[Math: x^2 + y^2 + 2fx + 2gy + c = 0]	[Math: (-f, -g)]	[Math: \sqrt{f^2 + g^2 - c}]

The tangent at a point on a circle is perpendicular to the radius at that point.

B. Vectors

For $\mathbf{v} = \begin{pmatrix} x \\ y \end{pmatrix}$:

$$|\mathbf{v}| = \sqrt{x^2 + y^2} \qquad \hat{\mathbf{v}} = \frac{\mathbf{v}}{|\mathbf{v}|}$$

Dot product:

$$\begin{pmatrix} a \\ b \end{pmatrix} \cdot \begin{pmatrix} c \\ d \end{pmatrix} = ac + bd$$

Angle between vectors:

$$\cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$$

- $\mathbf{a} \cdot \mathbf{b} = 0$: vectors are perpendicular.
- $\mathbf{b} = k\mathbf{a}$ for some scalar k : vectors are parallel.
- Three points A , B , C are collinear if $\overrightarrow{AB} = k\overrightarrow{AC}$.

Direction of a vector: $\theta = \arctan\left(\frac{y}{x}\right)$, then adjust for the correct quadrant (add $180°$ in Q2 or Q3).

C. Trigonometry

Conversion:

$$\theta° \times \frac{\pi}{180} \to \text{rad} \qquad \theta \text{ rad} \times \frac{180}{\pi} \to °$$

Arc and sector(θ in radians):

$$l = r\theta \qquad A_{\text{sector}} = \frac{1}{2}r^2\theta \qquad A_{\text{segment}} = \frac{1}{2}r^2(\theta - \sin\theta)$$

Exact values:

[Math: \theta]	[Math: 0]	[Math: 30^\circ]	[Math: 45^\circ]	[Math: 60^\circ]	[Math: 90^\circ]
[Math: \sin\theta]	[Math: 0]	[Math: \dfrac{1}{2}]	[Math: \dfrac{\sqrt{2}}{2}]	[Math: \dfrac{\sqrt{3}}{2}]	[Math: 1]
[Math: \cos\theta]	[Math: 1]	[Math: \dfrac{\sqrt{3}}{2}]	[Math: \dfrac{\sqrt{2}}{2}]	[Math: \dfrac{1}{2}]	[Math: 0]
[Math: \tan\theta]	[Math: 0]	[Math: \dfrac{1}{\sqrt{3}}]	[Math: 1]	[Math: \sqrt{3}]	undef.

CAST rule: All (Q1), Sine (Q2), Tangent (Q3), Cosine (Q4) are positive.

Graph properties:

Function	Period	Amplitude
[Math: y = a\sin(kx)]	[Math: \dfrac{2\pi}{k}]	[Math: \lvert a \rvert]
[Math: y = a\cos(kx)]	[Math: \dfrac{2\pi}{k}]	[Math: \lvert a \rvert]
[Math: y = a\tan(kx)]	[Math: \dfrac{\pi}{k}]	—

Pythagorean identities:

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta$$

Compound angle formulas:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Double angle formulas:

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Section 3: Introductory Calculus

A. Differentiation

[Math: f(x)]	[Math: f'(x)]
[Math: x^n]	[Math: nx^{n-1}]
[Math: \sin(ax)]	[Math: a\cos(ax)]
[Math: \cos(ax)]	[Math: -a\sin(ax)]

Chain rule(let $u = g(x)$, $y = f(u)$):

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Product rule($y = uv$):

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

Quotient rule($y = \frac{u}{v}$):

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Stationary points:set $f'(x) = 0$, solve for x, find y.

Second derivative at stationary point	Nature
[Math: f''(x) > 0]	Minimum

[Math: f''(x) < 0]	Maximum
[Math: f''(x) = 0]	Inconclusive; use sign-change test on [Math: f'(x)]

Tangent and normalat $(x0, y0)$: tangent gradient $mT = f'(x0)$; normal gradient $mN = -\frac{1}{mT}$.

Connected rates: $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$

B. Integration

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c \qquad (n \neq -1)$$

$$\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$$

$$\int \sin(ax) \, dx = -\frac{1}{a}\cos(ax) + c \qquad \int \cos(ax) \, dx = \frac{1}{a}\sin(ax) + c$$

Area under a curvebetween $x = a$ and $x = b$:

$$A = \int_a^b y \, dx$$

Area between two curves($f(x) \geq g(x)$ on $[a, b]$):

$$A = \int_a^b \left[f(x) - g(x) \right] dx$$

Volume of revolutionabout the x-axis:

$$V = \pi \int_a^b y^2 \, dx$$

Kinematics

Displacement s, velocity v, acceleration a, time t:

$$v = \frac{ds}{dt} \qquad a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

$$s = \int v\,dt \qquad v = \int a\,dt$$

SUVAT equations(uniform acceleration only; u = initial velocity, s = displacement):

$$v = u + at \qquad s = ut + \tfrac{1}{2}at^2 \qquad v^2 = u^2 + 2as \qquad s = \tfrac{1}{2}(u+v)t$$

Particle is stationary when $v = 0$. Direction changes when v changes sign.

Section 4: Probability and Statistics

A. Data Representation and Analysis

Mean:

$$\bar{x} = \frac{\sum x}{n} \quad \text{(ungrouped)} \qquad \bar{x} = \frac{\sum fx}{\sum f} \quad \text{(grouped)}$$

Variance and standard deviation:

$$S^2 = \frac{\sum (x-\bar{x})^2}{n} \qquad S = \sqrt{\frac{\sum (x-\bar{x})^2}{n}} \quad \text{(ungrouped)}$$

$$S^2 = \frac{\sum f(x-\bar{x})^2}{\sum f} \quad \text{(grouped)}$$

Measures of spread:

$$\text{Range} = x_{\max} - x_{\min} \qquad \text{IQR} = Q_3 - Q_1 \qquad \text{Semi-IQR} = \frac{Q_3 - Q_1}{2}$$

Skewness:

Box plot pattern	Skew	Order of averages
Right whisker longer / [Math: Q3 - Q2 > Q2 - Q1]	Positive	Mode < Median < Mean
Left whisker longer / [Math: Q3 - Q2 < Q2 - Q1]	Negative	Mean < Median < Mode
Symmetric	None	Mode = Median = Mean

B. Probability Theory

$$P(A) = \frac{\text{favourable outcomes}}{\text{total outcomes}} \qquad P(A') = 1 - P(A) \qquad 0 \le P(A) \le 1$$

Rule	Formula
Addition (general)	[Math: P(A \cup B) = P(A) + P(B) - P(A \cap B)]
Mutually exclusive ([Math: A \cap B = \emptyset])	[Math: P(A \cup B) = P(A) + P(B)]
Conditional	[Math: P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}]
Independent	[Math: P(A \cap B) = P(A) \cdot P(B)]

Two non-zero-probability events cannot be simultaneously mutually exclusive and independent.

Tree diagrams: multiply along branches for intersection probabilities; add across branches for union. The syllabus restricts trees to two initial branches.

Venn diagrams:restricted to two sets; the four regions (A only, B only, $A \cap B$, neither) must sum to $P(S) = 1$.