Inequalities

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Inequalities

The CSEC syllabus focuses on two types of inequality beyond the linear work covered in General Mathematics: **quadratic inequalities** and **rational inequalities** with linear factors. Both are solved by the same underlying strategy: find critical values, then determine the sign of the expression in each region.

The Key Idea: Sign Analysis

An inequality like $f(x) > 0$ asks where the expression $f(x)$ is positive. The sign of $f(x)$ can only change at points where $f(x) = 0$ or where $f(x)$ is undefined. These are the

critical values

. Between consecutive critical values, the sign is constant, so you only need to test one point per region.

Quadratic Inequalities

Method:

- 1. Rearrange so the right side is zero.
- 2. Solve $ax^2 + bx + c = 0$ to find the **critical values** (the roots of the equality).
- 3. State the critical values, then sketch the parabola with those roots marked on the x-axis. Shade the region the inequality describes.
- 4. Read off the solution and write it in set builder notation.

For $a > 0$ (upward parabola) with critical values $\alpha < \beta$

: the expression is negative between the roots and positive outside them. For $a < 0$

the regions reverse.

📋 **Example**

Solve $x^2 - x - 6 < 0$.

Solve: $(x-3)(x+2)=0$

Critical values: $x=-2$, $x=3$

Sketch: parabola opens upward ($a=1>0$). It lies below the x-axis between the roots, so shade that region.

$$\{x \in \mathbb{R} : -2 < x < 3\}$$

📋 **Example**

Solve $x^2 - 9 \geq 0$.

Solve: $(x-3)(x+3)=0$

Critical values: $x=-3$, $x=3$

Sketch: parabola opens upward. It lies above (or on) the x-axis outside the roots, so shade those two outer regions. Endpoints are included because the inequality is $\geq$.

$$\{x \in \mathbb{R} : x \leq -3\} \cup \{x \in \mathbb{R} : x \geq 3\}$$

Using a Sign Diagram

A sign diagram tracks the sign of each factor across every region. It is especially clear when the expression is already in factorised form.

📋 **Example**

Solve $(x+1)(x-4) \leq 0$.

Critical values: $x = -1$, $x = 4$

Region	$x+1$	$x-4$	Product
$x < -1$	$-$	$-$	$+$
$-1 < x < 4$	$+$	$-$	$-$
$x > 4$	$+$	$+$	$+$

The product is ≤ 0
in the middle region. Both endpoints satisfy the equality, so they are included.

$$\{x \in \mathbb{R} : -1 \leq x \leq 4\}$$

📝 **Exam Tip**

Never multiply both sides of an inequality by an expression containing x
to clear a fraction. The sign of that expression depends on x
, so the inequality direction may flip unpredictably. Use a sign diagram instead.

Rational Inequalities with Linear Factors

The syllabus specifies inequalities of the form $\frac{ax+b}{cx+d} > 0$ (or ≥ 0, < 0, ≤ 0).

Two types of critical value arise:

- Where the **numerator is zero**
: the fraction equals zero. Include this point when the inequality is $\leq$ or $\geq$
; exclude it for strict inequalities.
- Where the **denominator is zero**: the fraction is undefined. Always exclude this point.

Method: find both critical values, draw a sign diagram for numerator and denominator separately, combine signs, then write the solution in set builder notation.

📋 **Example**

Solve $\frac{x-1}{x+2} > 0$.

Critical values: $x = 1$ (numerator zero) and $x = -2$ (denominator zero, always excluded)

Region	$x-1$	$x+2$	Fraction
$x < -2$	$-$	$-$	$+$
$-2 < x < 1$	$-$	$+$	$-$
$x > 1$	$+$	$+$	$+$

The fraction is positive for $x < -2$ or $x > 1$. The inequality is strict ($>$), so $x = 1$ is excluded. $x = -2$ is always excluded (undefined).

$$\{x \in \mathbb{R} : x < -2\} \cup \{x \in \mathbb{R} : x > 1\}$$

📋 **Example**

Solve $\frac{2x+3}{x-5} \leq 0$.

Critical values: $x = -\frac{3}{2}$ (numerator zero) and $x = 5$ (denominator zero, always excluded)

Region	$2x+3$	$x-5$	Fraction
$x < -\frac{3}{2}$	$-$	$-$	$+$
$-\frac{3}{2} < x < 5$	$+$	$-$	$-$
$x > 5$	$+$	$+$	$+$

The fraction is ≤ 0 in the middle region. Include $x = -\frac{3}{2}$ (numerator zero with $\leq$) but exclude $x = 5$ (denominator always excluded).

$$\left\{x \in \mathbb{R} : -\frac{3}{2} \leq x < 5\right\}$$

Notation Reference

Set builder notation (standard form for exam solutions):

Situation	Set builder form
Connected range, strict	[Math: \lbrace x \in \mathbb{R} : a < x < b \rbrace]
Connected range, closed	[Math: \lbrace x \in \mathbb{R} : a \leq x \leq b \rbrace]
Mixed endpoints	[Math: \lbrace x \in \mathbb{R} : a \leq x < b \rbrace]
Two separate ranges	[Math: \lbrace x \in \mathbb{R} : x < a \rbrace \cup \lbrace x \in \mathbb{R} : x > b \rbrace]

The symbol $\in$ means "is a member of" and $\mathbb{R}$ denotes the set of real numbers.

Interval notation (alternative, also acceptable):

Notation	Meaning
[Math: (a, b)]	[Math: a < x < b]
[Math: [a, b]]	[Math: a \leq x \leq b]
[Math: [a, b)]	[Math: a \leq x < b]
[Math: (-\infty, a)]	[Math: x < a]
[Math: A \cup B]	[Math: x] satisfies [Math: A] or [Math: B]

🧠 **Remember**

Endpoint inclusion: a critical value is included (closed boundary, $\leq$ or $\geq$
) when the inequality is not strict and the expression is defined there. It is excluded (open boundary) when the inequality is strict, or when it is a denominator-zero point (fraction undefined).