Trigonometry

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Trigonometry

Trigonometry is the heaviest Paper 01 topic (9 of 45 items) and a recurring element of Paper 02. The section builds from exact values and graphs through to identity proofs and equation solving, all using radians as the default angle unit.

Radians

One **radian** is the angle subtended at the centre of a circle when the arc length equals the radius.

$$180^\circ = \pi \text{ radians}$$

Degrees to radians: multiply by $\frac{\pi}{180}$

Radians to degrees: multiply by $\frac{180}{\pi}$

Degrees	[-Math: 0°]	[-Math: 30°]	[-Math: 45°]	[-Math: 60°]	[-Math: 90°]	[-Math: 120°]	[-Math: 135°]	[-Math: 150°]	[-Math: 180°]	[-Math: 270°]	[-Math: 360°]
Radians	[-Math: 0]	[-Math: \frac{\pi}{6}]	[-Math: \frac{\pi}{4}]	[-Math: \frac{\pi}{3}]	[-Math: \frac{\pi}{2}]	[-Math: \frac{2\pi}{3}]	[-Math: \frac{3\pi}{4}]	[-Math: \frac{5\pi}{6}]	[-Math: \pi]	[-Math: \frac{3\pi}{2}]	[-Math: 2\pi]

Arc Length and Sector Area

For a circle of radius r and sector angle θ in **radians**:

$$\text{Arc length:} \quad s = r\theta \qquad \text{Sector area:} \quad A = \frac{1}{2}r^2\theta$$

📝 **Exam Tip**
Both formulas require θ in radians. Convert degrees before substituting. A common error is substituting degrees directly into $s = r\theta$.

Exact Trigonometric Values

These must be memorised. They derive from the equilateral triangle (for $30^\circ, 60^\circ$) and the isosceles right triangle (for 45°).

[Math: \theta]	[Math: \sin\theta]	[Math: \cos\theta]	[Math: \tan\theta]
[Math: 0]	[Math: 0]	[Math: 1]	[Math: 0]
[Math: \frac{\pi}{6}] [Math: (30°)]	[Math: \frac{1}{2}]	[Math: \frac{\sqrt{3}}{2}]	[Math: \frac{1}{\sqrt{3}}]
[Math: \frac{\pi}{4}] [Math: (45°)]	[Math: \frac{\sqrt{2}}{2}]	[Math: \frac{\sqrt{2}}{2}]	[Math: 1]
[Math: \frac{\pi}{3}] [Math: (60°)]	[Math: \frac{\sqrt{3}}{2}]	[Math: \frac{1}{2}]	[Math: \sqrt{3}]
[Math: \frac{\pi}{2}] [Math: (90°)]	[Math: 1]	[Math: 0]	undefined

For related angles in other quadrants, use the CAST diagram.

The CAST Diagram

The sign of each trig function depends on the quadrant. CAST tells you which functions are **positive** in each quadrant (all others are negative):

Quadrant	Degrees	Positive functions
I (first)	[Math: 0°] to [Math: 90°]	All: sin, cos, tan
II (second)	[Math: 90°] to [Math: 180°]	Sin only
III (third)	[Math: 180°] to [Math: 270°]	Tan only
IV (fourth)	[Math: 270°] to [Math: 360°]	Cos only

Related angle method: For any angle θ outside $[0^\circ, 90^\circ]$

, find the reference angle (acute angle to the nearest x
-axis), evaluate the trig function for that acute angle, then apply the sign from CAST.

📋 **Example**

Find $\sin 150^\circ$.

150° is in Quadrant II (sin positive). Reference angle: $180^\circ - 150^\circ = 30^\circ$.

$\sin 150^\circ = +\sin 30^\circ = \frac{1}{2}$

📋 **Example**

Find $\cos\left(\frac{5\pi}{6}\right)$.

$\frac{5\pi}{6} = 150^\circ$, Quadrant II (cos negative). Reference angle: 30° .

$\cos\left(\frac{5\pi}{6}\right) = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$

Graphs of Trigonometric Functions

For $y = \sin kx$, $y = \cos kx$, and $y = \tan kx$:

Property	[Math: \sin kx] / [Math: \cos kx]	[Math: \tan kx]
Amplitude	[Math: 1]	undefined
Period	[Math: \frac{2\pi}{k}]	[Math: \frac{\pi}{k}]
Range	[Math: [-1,\ 1]]	[Math: \mathbb{R}]

Increasing k compresses the graph horizontally (shorter period). The $\sin$ and $\cos$ graphs are identical in shape; $\cos$ is $\sin$ shifted $\frac{\pi}{2}$ to the left.

For $k = 2$ specifically: $y = \sin 2x$ and $y = \cos 2x$ each complete two full cycles in $[0, 2\pi]$
, so their period is π . This is the most commonly tested value of k beyond $k = 1$.

Trigonometric Identities

The fundamental **Pythagorean identity** derives from the unit circle:

$$\sin^2\theta + \cos^2\theta \equiv 1$$

Dividing through by $\cos^2\theta$:

$$\tan^2\theta + 1 \equiv \sec^2\theta$$

The identity $\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$ is equally essential.

Compound-Angle Formulas

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

Note the sign pattern: in $\cos(A+B)$ the right-hand side uses **minus**, and in $\cos(A-B)$ it uses **plus**

(opposite to the left-hand side). Students are not expected to prove these formulas, but must apply them accurately.

📋 **Example**

Find the exact value of $\sin 75^\circ$.

$$\sin 75^\circ = \sin(45^\circ + 30^\circ) = \sin 45^\circ\cos 30^\circ + \cos 45^\circ\sin 30^\circ$$

$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

Double-Angle Formulas

Setting $B = A$ in the compound-angle formulas gives:

$$\sin 2A = 2\sin A\cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

The three forms of $\cos 2A$ are all equivalent (each comes from substituting $\sin^2 A + \cos^2 A = 1$ into the first). Which to use depends on what the question asks for: if you need $\cos 2A$ in terms of $\sin A$ only, use $1 - 2\sin^2 A$; in terms of $\cos A$ only, use $2\cos^2 A - 1$; if both appear, the first form $\cos^2 A - \sin^2 A$ is most flexible.

Proving Trigonometric Identities

An **identity** is true for all valid values of the variable. To prove it, work on **one side only** (usually the more complex side) and transform it step by step until it matches the other side. The reason you cannot move terms across the equals sign is that doing so would assume the identity is already true before you have proved it, which is circular reasoning. The goal is to show both sides are equal independently.

📋 **Example**

Prove that $\frac{\sin 2\theta}{1 + \cos 2\theta} \equiv \tan\theta$.

Work on the left side:

$$\frac{\sin 2\theta}{1 + \cos 2\theta} = \frac{2\sin\theta\cos\theta}{1 + (2\cos^2\theta - 1)} = \frac{2\sin\theta\cos\theta}{2\cos^2\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta \quad ✓$$

📝 **Exam Tip**

Identity proofs almost always require substituting double-angle formulas and then simplifying. Look for opportunities to cancel or factor. If you are stuck, try expressing everything in terms of
$\sin$ and $\cos$.

Solving Trigonometric Equations

The syllabus requires solutions in $0 \leq \theta \leq 2\pi$ (radians). General solutions are not required.

Standard method:

- 1. Isolate the trig function (e.g. $\sin\theta = \frac{1}{2}$).
- 2. Find the reference angle using exact values or a calculator.
- 3. Use CAST to identify which quadrants give solutions in the required range.
- 4. List all solutions.

📋 **Example**

Solve $\cos\theta = -\frac{\sqrt{3}}{2}$ for $0 \leq \theta \leq 2\pi$.

Reference angle: $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$.

Cosine is negative in Quadrants II and III.

Quadrant II: $\theta = \pi - \dfrac{\pi}{6} = \dfrac{5\pi}{6}$

Quadrant III: $\theta = \pi + \dfrac{\pi}{6} = \dfrac{7\pi}{6}$

Equations Using Identities

Many trig equations require substituting an identity to reduce to a standard form.

📋 **Example**

Solve $2\cos^2\theta - 1 = 0$ for $0 \le \theta \le 2\pi$.

Recognise $2\cos^2\theta - 1 = \cos 2\theta$, but it is simpler to solve directly:

$$\cos^2\theta = \frac{1}{2} \Rightarrow \cos\theta = \pm\frac{\sqrt{2}}{2}$$

Reference angle: $\frac{\pi}{4}$. Cosine is positive in I, IV and negative in II, III:

$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

📋 **Example**

Solve $\sin 2\theta = \sin\theta$ for $0 \le \theta \le 2\pi$.

$$2\sin\theta\cos\theta = \sin\theta \Rightarrow \sin\theta(2\cos\theta - 1) = 0$$

Either $\sin\theta = 0$: $\theta = 0, \pi, 2\pi$.

Or $\cos\theta = \frac{1}{2}$: $\theta = \frac{\pi}{3}, \frac{5\pi}{3}$.

Solutions: $\theta = 0,\ \frac{\pi}{3},\ \pi,\ \frac{5\pi}{3},\ 2\pi$.

🧠 **Remember**

In the equation above, $\sin\theta = 0$ gives $\theta = 0$, π, and 2π within $[0, 2\pi]$. Including the endpoints of a closed interval is required. Never automatically exclude 0 or 2π.