Algebraic Expressions

Matthew Williams • Math • May 6, 2026

Algebra is a language for describing patterns before you know the exact numbers. A letter such as x or n lets you write one rule that works for many possible values.

CSEC tests algebra at all three cognitive levels: knowing symbols, simplifying expressions correctly, and modelling word problems. Each example shows what a symbol represents and why each operation is used.

Symbolic Representation

Symbols let you describe a general situation. Before simplifying or solving, decide what each letter stands for; otherwise the expression becomes a string of signs instead of a statement with meaning.

In algebra, we use letters to represent numbers:

- x, y, a, b = variables (unknown quantities)
- $3x$ = 3 times some number
- $x + 5$ = some number plus 5
- $2a - b$ = twice a, minus b

Translating Between Words and Algebra

Word problems become easier when you translate one phrase at a time. Look for operation words: "sum" suggests addition, "difference" suggests subtraction, "product" suggests multiplication, and "quotient" suggests division.

Words and algebra

. Word phrase ’algebraic expression

- "A number increased by 7" ’ $x+7$
- "Twice a number minus 3" ’ $2x-3$
- "The sum of two consecutive numbers" ’ $x+(x+1)=2x+1$
- "The product of a number and 5" ’ $5x$
- "A number divided by 2" ’ $\frac{x}{2}$

. Algebraic expression ’word phrase

- $3a+2$ ’"Three times a , plus 2"
- $\frac{y}{4}$ ’"A quarter of y "
- $5-x$ ’"5 minus a number"

Directed Numbers (Signed Numbers)

Directed numbers show direction from zero. A negative sign may mean debt, below sea level, loss, reverse movement, or temperature below zero, depending on the context.

Positive numbers are greater than zero. **Negative** numbers are less than zero.

$$(-3)+(+5)=2$$

$$(-3)+(-5)=-8$$

$$(-3)\times(-5)=15$$

Rules:

- **Same signs multiply to positive** : $(+)\times(+)=(+)$ and $(-)\times(-)=(+)$
- **Different signs multiply to negative** : $(+)\times(-)=(-)$

📋 **Example**

$(-4)\times(-7)=28$ (both negative ’positive)

$(-4)\times 7=-28$ (different signs ’negative)

$(-3)-(-8)=-3+8=5$ (subtract negative = add)

Operations with Algebraic Expressions

Adding and Subtracting Like Terms

Like terms can be combined because they count the same kind of object. Three x's plus five x's make eight x's, but three x's plus five y's must stay separate.

Like terms have the same variable and power.

$$3x + 5x = 8x$$

$$2a^2 + 3a^2 = 5a^2$$

$$3x + 5y \text{ cannot be combined (different variables)}$$

📋 **Example**

Simplify: $4x + 3y - 2x + 5y$

Group like terms: $(4x - 2x) + (3y + 5y) = 2x + 8y$

Multiplication and Division of Expressions

Multiplication spreads a factor across every term in a bracket. Division does the reverse by sharing each term by the same divisor.

Multiply: Use the distributive law $a(b + c) = ab + ac$

$$2 \times (x + 3) = 2x + 6$$

$$x \times (x + 5) = x^2 + 5x$$

Divide: Divide each term by the divisor

$$\frac{6x + 9}{3} = \frac{6x}{3} + \frac{9}{3} = 2x + 3$$

Expanding Brackets

Expanding removes brackets by multiplying. It is useful when an expression must be simplified or compared with another expression in standard form.

Distributive law: $a(b + c) = ab + ac$

📋 **Example**

Single bracket:

$$3(2x + 5) = 6x + 15$$

$$x(x - 4) = x^2 - 4x$$

Double brackets: $(a + b)(c + d) = ac + ad + bc + bd$

📋 **Example**

$$(x + 3)(x + 5)$$

Using FOIL (First, Outer, Inner, Last):

- **F** irst: $x \times x = x^2$
- **O** uter: $x \times 5 = 5x$
- **I** nner: $3 \times x = 3x$
- **L** ast: $3 \times 5 = 15$

$$x^2 + 5x + 3x + 15 = x^2 + 8x + 15$$

📋 **Example**

$$(2x - 3)(x + 4)$$

$$= 2x(x) + 2x(4) + (-3)(x) + (-3)(4)$$

$$= 2x^2 + 8x - 3x - 12$$

$$= 2x^2 + 5x - 12$$

Substitution and Binary Operations

Substitution

Substitution turns a general expression into a specific value. Brackets are important because they preserve signs, especially when substituting negative numbers.

Substitution means: replace the letter with the number you're given, then calculate.

Why this matters: Many algebra problems ask "What happens if x equals this number?" Substitution lets you find the answer by plugging in the value.

The Three-Step Process

Step 1: Identify which variable is being replaced and what value it gets

Step 2: Replace every instance of that letter with the value (use brackets to avoid confusion)

Step 3: Follow the order of operations (PEMDAS: Powers, Multiplication/Division, Addition/Subtraction)

Example 1: Simple Expression

If $x = 3$, find $2x^2 + 5x - 1$:

Step 1: We're replacing x with 3

Step 2: Write with brackets: $2(3)^2 + 5(3) - 1$

Step 3: Calculate carefully in order:

- Powers first: $(3)^2 = 9$
- Then: $2(9) + 5(3) - 1 = 18 + 15 - 1$
- Finally: $18 + 15 - 1 = 32$

Answer: 32

Example 2: Multiple Variables

If $a = 2, b = 3$**, find** $3a^2 + 2ab - b$**:**

Step 1: Replace a with 2 and b with 3

Step 2: Write with brackets: $3(2)^2 + 2(2)(3) - (3)$

Step 3: Calculate in order:

- Powers: $(2)^2 = 4$
- Multiplication: $3(4) = 12$, $2(2)(3) = 12$
- Addition and subtraction: $12 + 12 - 3 = 21$

Answer: 21

Key tip: When you have multiple variables, be EXTRA careful to replace each one correctly.

When Substitution Goes Wrong

🧠 **Remember**

Common mistakes:

- Forgetting PEMDAS order (doing addition before powers)
- Not replacing ALL instances of the variable
- Arithmetic errors in the calculation step

Check yourself: Substitute your answer back and make sure the math works!

Binary Operations

For binary operations, the symbol is only a label for a rule. Do not assume it behaves like ordinary multiplication or addition unless the definition says so.

A **binary operation** is a special rule that tells you how to combine two numbers. It's not just the normal operations (+, −, ×, ÷), it can be anything!

Why learn this? Tests use binary operations to check if you can follow instructions and substitute correctly. It's a skill test, not a math test.

Reading a Binary Operation Definition

When you see "$a * b = 2a + b$", it means:

- $*$ is the special symbol (could be anything: , ; Æetc.)
- a is the FIRST number

- b is the SECOND number
- The rule is: multiply the first number by 2, then add the second number

📋 **Example**

The operation $ab = 2a + b$ **Find** 35**:**

Step 1: Identify: $a = 3$, $b = 5$

Step 2: Use the rule $2a + b$:

$$3 * 5 = 2(3) + 5 = 6 + 5 = 11$$

Answer: 11

Nested Operations (Operations Inside Operations)

When you see $2(31)$, solve the parentheses FIRST.

📋 **Example**

Find $2(31)$ **where** $a * b = 2a + b$**:**

Step 1: Do the inner operation first: $3 * 1$

$$3 * 1 = 2(3) + 1 = 6 + 1 = 7$$

Step 2: Now use that result: $2 * 7$

$$2 * 7 = 2(2) + 7 = 4 + 7 = 11$$

Answer: 11

Important: Order matters! $(ab)c$ is NOT the same as $a(bc)$
(these operations aren't "associative")