

Algebraic Expressions

Matthew Williams • Math • May 6, 2026

Algebra is a language for describing patterns before you know the exact numbers. A letter such as

x or n lets you write one rule that works for many possible values.

CSEC tests algebra at all three cognitive levels: knowing symbols, simplifying expressions correctly, and modelling word problems. Each example shows what a symbol represents and why each operation is used.

Symbolic Representation

Symbols let you describe a general situation. Before simplifying or solving, decide what each letter stands for; otherwise the expression becomes a string of signs instead of a statement with meaning.

In algebra, we use letters to represent numbers:

- x , y , a , b = variables (unknown quantities)
- $3x$ = 3 times some number
- $x + 5$ = some number plus 5
- $2a - b$ = twice a , minus b

Translating Between Words and Algebra

Word problems become easier when you translate one phrase at a time. Look for operation words: "sum" suggests addition, "difference" suggests subtraction, "product" suggests multiplication, and "quotient" suggests division.

Words and algebra

1. Word phrase 'algebraic expression

- "A number increased by 7" ' $x + 7$
- "Twice a number minus 3" ' $2x - 3$
- "The sum of two consecutive numbers" ' $x + (x + 1) = 2x + 1$
- "The product of a number and 5" ' $5x$
- "A number divided by 2" ' $\frac{x}{2}$

2. Algebraic expression 'word phrase

- $3a + 2$ "Three times a , plus 2"
- $\frac{y}{4}$ "A quarter of y "
- $5 - x$ "5 minus a number"

Directed Numbers (Signed Numbers)

Directed numbers show direction from zero. A negative sign may mean debt, below sea level, loss, reverse movement, or temperature below zero, depending on the context.

Positive numbers are greater than zero. **Negative** numbers are less than zero.

$$(-3) + (+5) = 2$$

$$(-3) + (-5) = -8$$

$$(-3) \times (-5) = 15$$

Rules:

- **Same signs multiply to positive** : $(+) \times (+) = (+)$ and $(-) \times (-) = (+)$
- **Different signs multiply to negative** : $(+) \times (-) = (-)$

Example

$$(-4) \times (-7) = 28 \text{ (both negative 'positive)}$$

$$(-4) \times 7 = -28 \text{ (different signs 'negative)}$$

$$(-3) - (-8) = -3 + 8 = 5 \text{ (subtract negative = add)}$$

Operations with Algebraic Expressions

Adding and Subtracting Like Terms

Like terms can be combined because they count the same kind of object. Three x 's plus five x 's make eight x 's, but three x 's plus five y 's must stay separate.

Like terms have the same variable and power.

$$3x + 5x = 8x$$

$$2a^2 + 3a^2 = 5a^2$$

$3x + 5y$ cannot be combined (different variables)

Example

Simplify: $4x + 3y - 2x + 5y$

Group like terms: $(4x - 2x) + (3y + 5y) = 2x + 8y$

Multiplication and Division of Expressions

Multiplication spreads a factor across every term in a bracket. Division does the reverse by sharing each term by the same divisor.

Multiply: Use the distributive law $a(b + c) = ab + ac$

$$2 \times (x + 3) = 2x + 6$$

$$x \times (x + 5) = x^2 + 5x$$

Divide: Divide each term by the divisor

$$\frac{6x + 9}{3} = \frac{6x}{3} + \frac{9}{3} = 2x + 3$$

Expanding Brackets

Expanding removes brackets by multiplying. It is useful when an expression must be simplified or compared with another expression in standard form.

Distributive law: $a(b + c) = ab + ac$

Example

Single bracket:

$$3(2x + 5) = 6x + 15$$

$$x(x - 4) = x^2 - 4x$$

Double brackets: $(a + b)(c + d) = ac + ad + bc + bd$

Example

$$(x + 3)(x + 5)$$

Using FOIL (First, Outer, Inner, Last):

- **F** irst: $x \times x = x^2$
- **O** uter: $x \times 5 = 5x$
- **I** nner: $3 \times x = 3x$
- **L** ast: $3 \times 5 = 15$

$$x^2 + 5x + 3x + 15 = x^2 + 8x + 15$$

Example

$$(2x - 3)(x + 4)$$

$$= 2x(x) + 2x(4) + (-3)(x) + (-3)(4)$$

$$= 2x^2 + 8x - 3x - 12$$

$$= 2x^2 + 5x - 12$$

Substitution and Binary Operations

Substitution

Substitution turns a general expression into a specific value. Brackets are important because they preserve signs, especially when substituting negative numbers.

Substitution means: replace the letter with the number you're given, then calculate.

Why this matters: Many algebra problems ask "What happens if x equals this number?" Substitution lets you find the answer by plugging in the value.

The Three-Step Process

Step 1: Identify which variable is being replaced and what value it gets

Step 2: Replace every instance of that letter with the value (use brackets to avoid confusion)

Step 3: Follow the order of operations (PEMDAS: Powers, Multiplication/Division, Addition/Subtraction)

Example 1: Simple Expression

If $x = 3$, find $2x^2 + 5x - 1$:

Step 1: We're replacing x with 3

Step 2: Write with brackets: $2(3)^2 + 5(3) - 1$

Step 3: Calculate carefully in order:

- Powers first: $(3)^2 = 9$
- Then: $2(9) + 5(3) - 1 = 18 + 15 - 1$
- Finally: $18 + 15 - 1 = 32$

Answer: 32

Example 2: Multiple Variables

If $a = 2$, $b = 3$, find $3a^2 + 2ab - b$:

Step 1: Replace a with 2 and b with 3

Step 2: Write with brackets: $3(2)^2 + 2(2)(3) - (3)$

Step 3: Calculate in order:

- Powers: $(2)^2 = 4$
- Multiplication: $3(4) = 12$, $2(2)(3) = 12$
- Addition and subtraction: $12 + 12 - 3 = 21$

Answer: 21

Key tip: When you have multiple variables, be EXTRA careful to replace each one correctly.

When Substitution Goes Wrong**Remember****Common mistakes:**

- Forgetting PEMDAS order (doing addition before powers)
- Not replacing ALL instances of the variable
- Arithmetic errors in the calculation step

Check yourself: Substitute your answer back and make sure the math works!

Binary Operations

For binary operations, the symbol is only a label for a rule. Do not assume it behaves like ordinary multiplication or addition unless the definition says so.

A **binary operation** is a special rule that tells you how to combine two numbers. It's not just the normal operations (+, -, ×, ÷), it can be anything!

Why learn this? Tests use binary operations to check if you can follow instructions and substitute correctly. It's a skill test, not a math test.

Reading a Binary Operation Definition

When you see " $a * b = 2a + b$ ", it means:

- $*$ is the special symbol (could be anything: , , etc.)
- a is the FIRST number

- b is the SECOND number
- The rule is: multiply the first number by 2, then add the second number

Example

The operation $ab = 2a + b$. Find 35:

Step 1: Identify: $a = 3$, $b = 5$

Step 2: Use the rule $2a + b$:

$$3 * 5 = 2(3) + 5 = 6 + 5 = 11$$

Answer: 11

Nested Operations (Operations Inside Operations)

When you see $2(31)$, solve the parentheses FIRST.

Example

Find $2(31)$ where $a * b = 2a + b$:

Step 1: Do the inner operation first: $3 * 1$

$$3 * 1 = 2(3) + 1 = 6 + 1 = 7$$

Step 2: Now use that result: $2 * 7$

$$2 * 7 = 2(2) + 7 = 4 + 7 = 11$$

Answer: 11

Important: Order matters! $(ab)c$ is NOT the same as $a(bc)$
(these operations aren't "associative")