Algebraic Fractions & Laws of Indices

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Algebraic fractions and indices are simplification tools. They help you rewrite complicated expressions into cleaner forms without changing their value.

In the CSEC exam, these skills often appear inside larger algebra questions rather than as a topic by itself. You may need to simplify before solving, factor before cancelling, or apply index laws before substituting. Each line should preserve equality, so explain what rule you are using when the step is not obvious.

Simplifying Algebraic Fractions

The safest way to simplify an algebraic fraction is to factor first, then cancel common factors. Cancelling terms that are being added is a serious algebra mistake.

Algebraic fractions work exactly like regular fractions, but with variables instead of just numbers.

The Key Principle: You can cancel something from top and bottom if it appears in BOTH places.

Rule 1: Cancel Common Variables

If the same variable appears on top and bottom, you can cross them out.

📋 **Example**

Simplify $\frac{6x}{9x}$:

Method: Break into two fractions

$$\frac{6x}{9x} = \frac{6}{9} \times \frac{x}{x}$$

Now: $\frac{6}{9} = \frac{2}{3}$ and $\frac{x}{x} = 1$

$$\frac{6x}{9x} = \frac{2}{3}$$

Why this works: $x \div x = 1$, just like $5 \div 5 = 1$

Rule 2: Factor the Numerator, Then Cancel

When the numerator is more complex, factor it first. Then look for common factors with the denominator.

📋 **Example**

Simplify $\frac{x^2+5x}{x}$ **:**

Step 1: Factor the numerator (top)

$$x^2 + 5x = x(x+5)$$

Why? Because both terms have an x: $x^2 = x \cdot x$ and $5x = 5 \cdot x$

Step 2: Rewrite the fraction

$$\frac{x^2+5x}{x} = \frac{x(x+5)}{x}$$

Step 3: Cancel the x from top and bottom

$$\frac{x(x+5)}{x} = x + 5$$

Check: If $x = 3$: Original = $\frac{9+15}{3} = \frac{24}{3} = 8$, Simplified = $3 + 5 = 8$

🧠 **Remember**

You can ONLY cancel if:

- The same thing appears in BOTH numerator and denominator
- The whole thing is being multiplied (not added)

Wrong: $\frac{x+2}{x}$, you CANNOT cancel the x here! (The x is only in one part of a sum)

Right: $\frac{x(x+2)}{x}$, now you CAN cancel (the x is being multiplied)

Adding and Subtracting Algebraic Fractions

Addition and subtraction depend on matching denominators because the pieces must be the same size before they can be combined.

Just like regular fractions, you need a **common denominator** to add or subtract.

Case 1: Same Denominator (Easy)

If the denominators match, just combine the numerators.

📋 **Example**

Add $\frac{2}{x} + \frac{3}{x}$:

Same denominator: x

$$\frac{2}{x} + \frac{3}{x} = \frac{2+3}{x} = \frac{5}{x}$$

(This is just like $\frac{2}{5} + \frac{3}{5} = \frac{5}{5} = 1$)

Case 2: Different Denominators (Harder)

Find the **Least Common Denominator (LCD)**, the smallest expression both denominators divide into.

📋 **Example**

Add $\frac{1}{x} + \frac{1}{y}$:

Step 1: What's the LCD? Since we have x and y, the LCD is xy

Step 2: Convert each fraction to use the LCD

- $\frac{1}{x} = \frac{1\times y}{x\times y} = \frac{y}{xy}$ (multiply top and bottom by y)
- $\frac{1}{y} = \frac{1\times x}{y\times x} = \frac{x}{xy}$ (multiply top and bottom by x)

Step 3: Now add them

$$\frac{y}{xy} + \frac{x}{xy} = \frac{y+x}{xy} = \frac{x+y}{xy}$$

Check: If $x = 2, y = 3$: Original = $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$, Simplified = $\frac{2+3}{2\times 3} = \frac{5}{6}$

Finding LCD Strategy

📝 **Exam Tip**

To find the LCD with variable denominators:

- 1. List all the factors that appear
- 2. Use each factor the maximum number of times it appears in any denominator

Example: denominators x^2, xy, y 'LCD is x^2y (use x twice, y once)

Laws of Indices (Exponents)

What Are Indices?

Indices are shorthand for repeated multiplication. They are not decoration; they control how many times the base is used as a factor.

Index (or **exponent**) is the small number that says "multiply this number by itself how many times."

$$x^3 = x \times x \times x \text{ (three times)}$$

$$2^4 = 2 \times 2 \times 2 \times 2 = 16 \text{ (four times)}$$

The index tells you how many times to use the base.

Rule 1: Multiply

Use this rule only when the bases are the same. $x^2 \times x^3$ can be combined, but $x^2 \times y^3$ cannot become one power because the bases are different.

When you multiply powers with the SAME base, ADD the exponents:

$$a^m \times a^n = a^{m+n}$$

Why? Because:

$$x^2 \times x^3 = (x \times x) \times (x \times x \times x) = x \times x \times x \times x \times x = x^5$$

You're multiplying 2 of them times 3 of them = 5 of them total.

📋 **Example**

$x^2 \times x^3$:

Same base (x), so add exponents: $2 + 3 = 5$

$$x^2 \times x^3 = x^5$$

📋 **Example**

$a^4 \times a^2 \times a^3$:

Add all exponents: $4 + 2 + 3 = 9$

$$a^4 \times a^2 \times a^3 = a^9$$

Rule 2: Divide

Division removes repeated factors. Subtracting the exponents counts how many copies of the base remain after cancellation.

When you divide powers with the SAME base, SUBTRACT the exponents:

$$\frac{a^m}{a^n} = a^{m-n}$$

Why? Because:

$$\frac{x^5}{x^2} = \frac{x \times x \times x \times x \times x}{x \times x} = x \times x \times x = x^3$$

Cancel two of them, and you have three left. $5 - 2 = 3$.

📋 **Example**

$\frac{y^5}{y^2}$:

Same base (y), so subtract exponents: $5 - 2 = 3$

$$\frac{y^5}{y^2} = y^3$$

📋 **Example**
$\frac{a^8}{a^3}$:

Subtract: $8 - 3 = 5$

$$\frac{a^8}{a^3} = a^5$$

Rule 3: Power of a Power

A power of a power means repeated groups of repeated factors. Multiplying the indices counts the total number of factors.

When you have a power raised to another power, MULTIPLY the exponents:

$$(a^m)^n = a^{mn}$$

Why? Because:

$$(x^2)^3 = x^2 \times x^2 \times x^2 = (x \times x) \times (x \times x) \times (x \times x) = x^6$$

You have 3 groups of x^2, so $2 \times 3 = 6$ x's total.

📋 **Example**
$(a^2)^3$:

Multiply exponents: $2 \times 3 = 6$

$$(a^2)^3 = a^6$$

📋 **Example**
$(y^4)^2$:

Multiply: $4 \times 2 = 8$

$$(y^4)^2 = y^8$$

Rule 4: Product Rule

When a whole product is raised to a power, every factor inside the brackets is repeated. This includes numerical coefficients, not only variables.

When you raise a product to a power, apply the power to EACH part:

$$(ab)^n = a^n b^n$$

Why? Because:

$$(xy)^3 = (xy) \times (xy) \times (xy) = (x \times x \times x) \times (y \times y \times y) = x^3 y^3$$

📋 **Example**

$(2x)^3$:

Apply the power to both: 2^3 and x^3

$$(2x)^3 = 2^3 \times x^3 = 8x^3$$

📋 **Example**

$(ab)^2$:

Apply to each: a^2 and b^2

$$(ab)^2 = a^2 b^2$$

Rule 5: Zero Power

The zero power rule often surprises students because it does not mean "nothing". It comes from a pattern of division where equal powers cancel completely.

Any number (except 0) raised to the power 0 equals 1:

$$a^0 = 1$$

Why? Think about the divide rule:

$$\frac{a^n}{a^n} = 1 \text{ (something divided by itself)}$$

But also:

$$\frac{a^n}{a^n} = a^{n-n} = a^0$$

So $a^0 = 1$.

📋 **Example**

$5^0 = 1$

$x^0 = 1$ **(for any** $x \neq 0$**)**

$(7xyz)^0 = 1$

No matter how complicated the base, if the power is 0, the answer is always 1.

Rule 6: Negative Powers

A negative exponent shows position, not negativity. It moves the factor to the other side of a fraction bar and makes the exponent positive.

A negative power means "put it in a fraction":

$$a^{-n} = \frac{1}{a^n}$$

Why? Using the divide rule:

$$\frac{a^0}{a^n} = a^{0-n} = a^{-n}$$

But $a^0 = 1$, so:

$$a^{-n} = \frac{1}{a^n}$$

📋 **Example**

2^{-2}:

Flip to a fraction with positive power:

$$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$$

📋 **Example**

x^{-3}:

Flip:

$$x^{-3} = \frac{1}{x^3}$$

📋 **Example**

$(2x)^{-1}$:

Flip:

$$(2x)^{-1} = \frac{1}{2x}$$

🧠 **Remember**

Negative power `negative answer!

$2^{-2} = \frac{1}{4}$ (positive!)

$-2^2 = -4$ (this is different, the negative is NOT an exponent)

All Six Rules Summary

Rule	Formula	Example	Why
Multiply	[Math: a^m \times a^n = a^{m+n}]	[Math: x^2 \times x^3 = x^5]	Counting total uses
Divide	[Math: \frac{a^m}{a^n} = a^{m-n}]	[Math: \frac{y^5}{y^2} = y^3]	Canceling
Power of Power	[Math: (a^m)^n = a^{mn}]	[Math: (a^2)^3 = a^6]	Nested multiplication
Product	[Math: (ab)^n = a^n b^n]	[Math: (2x)^3 = 8x^3]	Distributing power

Zero	[Math: a^0 = 1]	[Math: 5^0 = 1]	Self-division
Negative	[Math: a^{-n} = \frac{1}{a^n}]	[Math: 2^{-2} = \frac{1}{4}]	Reciprocal

📝 **Exam Tip**

Strategy for index problems:

- 1. Identify which rule applies (same base? power of power? product?)
- 2. Apply that rule
- 3. Simplify
- 4. Check whether multiple rules need to be combined

Example: $(2x^2)^3 \times x^{-4}$

- First: Apply product rule ' $2^3(x^2)^3 = 8x^6$
- Then: Multiply with x^{-4} ' $8x^6 \times x^{-4} = 8x^{6-4} = 8x^2$