

# Inequalities & Changing the Subject

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Inequalities and formula rearrangement both test whether you understand balance. With equations, both sides are equal; with inequalities, one side is larger or smaller. With formulae, the goal is to make a chosen variable stand alone so it can be calculated directly.

CSEC questions may ask for a solution set, a number-line representation, or a formula rewritten for a different subject. Write each operation clearly on both sides, and remember that multiplying or dividing an inequality by a negative reverses the sign. That one rule is a common source of lost marks.

## Solving Inequalities in One Unknown

Use the same methods as equations, BUT flip the sign when multiplying/dividing by negatives!

$$ax + b < c$$

### Example

**[Math:  $2x + 3 < 11$ ]**

[MathBlock]

$$2x < 8$$

[/MathBlock]

[MathBlock]


$$x < 4$$

[/MathBlock]

Solution: All numbers less than 4

<JSXGraph id="algebra-ineq-1" title="x < 4" boundingbox=[[-3, 1.5, 7, -1.5]] axis={false} height={140}

code="board.create('axis', [[-3,0],[7,0]],{ticks:{insertTicks:false,ticksDistance:1,minorTicks:0,majorHeight:8}});b  
/>

 **Example**

$$-3x + 2 \geq 8$$

[MathBlock]

$$-3x \geq 6$$

[/MathBlock]

**Flip the sign when dividing by negative!**

[MathBlock]

$$x \leq -2$$

[/MathBlock]

<JSXGraph id="algebra-ineq-2" title="x ≤ -2" boundingbox=[[-6, 1.5, 3, -1.5]] axis={false}

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/>

<MathVideo src="/media/mathematics/Animation6InequalityNegativeCoefficient.mp4"

title="Solving Inequalities with Negative Coefficients" />

 **Remember****INEQUALITY GOLDEN RULE:**

When you multiply or divide both sides by a **negative number**, FLIP the inequality sign!

$$[Math: 5 > 2], \text{ but } [Math: -5 < -2]$$

## Part 10: Changing the Subject of a Formula — Rearranging for Different Variables

### What Does "Subject" Mean?

The **subject** is the variable that's isolated on one side (usually the left) of the equals sign.

$$V = \pi r^2 h \text{ } \rightarrow \text{ } V \text{ is the subject}$$

**Changing the subject** means rearranging so a DIFFERENT variable is isolated.

$$r^2 = \frac{V}{\pi h} \text{ } \rightarrow \text{ } r^2 \text{ is now the subject}$$

## Why Changing the Subject Matters

Formulas come in different forms depending on what you need to find:

- $A = \pi r^2$  if you know  $r$  and want  $A$
- $r = \sqrt{\frac{A}{\pi}}$  if you know  $A$  and want  $r$

Both are the SAME formula, just rearranged for different purposes.

## Strategy: Treat It Like Solving an Equation

Rearranging is just like solving for a variable in an equation! Use **inverse operations** to isolate the target variable.

### Example 1: Linear Formula

#### Example

**Make  $x$  the subject of:  $y = 3x + 2$**

**Step 1:** Subtract 2 from both sides (undo addition)

$$y - 2 = 3x + 2 - 2$$

$$y - 2 = 3x$$


**Step 2:** Divide by 3 (undo multiplication)

$$\frac{y-2}{3} = \frac{3x}{3}$$

$$x = \frac{y-2}{3}$$

**Check:** If  $y = 8$ , then  $x = \frac{8-2}{3} = 2$ . Original:  $8 = 3(2) + 2 = 8$

### Example 2: Formula With Multiplication

 **Example**

**Make  $r$  the subject of:  $C = 2\pi r$**

This formula relates circumference ( $C$ ) to radius ( $r$ ).

**Step 1:** Identify what's attached to  $r$

- $r$  is being multiplied by  $2\pi$

**Step 2:** Divide both sides by  $2\pi$

[MathBlock]

$$\frac{C}{2\pi} = \frac{2\pi r}{2\pi}$$

[/MathBlock]

[MathBlock]

$$r = \frac{C}{2\pi}$$

[/MathBlock]

**Meaning:** If you know the circumference, divide by  $2\pi$  to find the radius.

### Example 3: Formula With Division

 **Example**

**Make  $h$  the subject of:  $A = \frac{1}{2}bh$**

This is the area formula for a triangle.

**Step 1:** Identify what's with  $h$

- $h$  is being multiplied by  $\frac{1}{2}b$

**Step 2:** Divide both sides by  $\frac{1}{2}b$  (or multiply by its reciprocal  $\frac{2}{b}$ )

[MathBlock]

$$\frac{A}{\frac{1}{2}b} = h$$

[/MathBlock]

Simplify:  $\frac{A}{\frac{1}{2}b} = \frac{2A}{b}$

[MathBlock]

$$h = \frac{2A}{b}$$

[/MathBlock]

**Meaning:** If you know area and base, use this to find height.

**Check:** If  $A = 20$  and  $b = 4$ , then  $h = \frac{2(20)}{4} = 10$ . Original:  $20 = \frac{1}{2}(4)(10) = 20$

## Example 4: Formula With Powers (Needs Square Root)

### Example

Make  $a$  the subject of:  $V = \frac{1}{3}a^2 h$

This is the volume formula for a pyramid.

**Step 1:** Multiply both sides by 3 (undo  $\frac{1}{3}$ )

[MathBlock]

$$3V = a^2 h$$

[/MathBlock]

**Step 2:** Divide both sides by  $h$  (undo multiplication by  $h$ )

[MathBlock]

$$\frac{3V}{h} = a^2$$

[/MathBlock]

**Step 3:** Take the square root (undo the square)

[MathBlock]

$$a = \sqrt{\frac{3V}{h}}$$

[/MathBlock]

Or written another way:  $a = \sqrt{\frac{3V}{h}} = \frac{\sqrt{3V}}{\sqrt{h}}$

**Meaning:** If you know volume and height, use this to find the base side  $a$ .

## Step-by-Step Strategy

- 1. **Identify the target variable** (the one you want to isolate)
- 2. **Work backwards through operations** using inverses
- 3. **Order matters:**
  - Undo addition/subtraction FIRST (loose operations)
  - Then undo multiplication/division
  - Then undo powers/roots
- 4. **Do the same operation to BOTH sides**

### Remember

#### Common mistakes:

- Forgetting to apply the operation to ALL terms
- Forgetting that when you divide by something, divide the ENTIRE other side
- Forgetting that  $\sqrt{x^2} = |x|$  (absolute value), but usually we just write  $x$