

Inequalities & Changing the Subject

Matthew Williams • Math • May 6, 2026

Inequalities and formula rearrangement both test whether you understand balance. With equations, both sides are equal; with inequalities, one side is larger or smaller. With formulae, the goal is to make a chosen variable stand alone so it can be calculated directly.

CSEC questions may ask for a solution set, a number-line representation, or a formula rewritten for a different subject. Write each operation clearly on both sides, and remember that multiplying or dividing an inequality by a negative reverses the sign. That one rule is a common source of lost marks.

Solving Inequalities in One Unknown

Use the same methods as equations, BUT flip the sign when multiplying/dividing by negatives!

$$ax + b < c$$

Example

$$2x + 3 < 11$$

$$2x < 8$$

$$x < 4$$

Solution: All numbers less than 4

Example

$$-3x + 2 \geq 8$$

$$-3x \geq 6$$

Flip the sign when dividing by negative!

$$x \leq -2$$

Remember**INEQUALITY GOLDEN RULE:**

When you multiply or divide both sides by a **negative number**, FLIP the inequality sign!

$$5 > 2, \text{ but } -5 < -2$$

Changing the Subject of a Formula

What Does "Subject" Mean?

The **subject** is the variable that's isolated on one side (usually the left) of the equals sign.

$$V = \pi r^2 h \leftarrow V \text{ is the subject}$$

Changing the subject means rearranging so a DIFFERENT variable is isolated.

$$r^2 = \frac{V}{\pi h} \leftarrow r^2 \text{ is now the subject}$$

Why Changing the Subject Matters

Formulae take different forms depending on which quantity is unknown:

- $A = \pi r^2$ if you know r and want A
- $r = \sqrt{\frac{A}{\pi}}$ if you know A and want r

Both are the SAME formula, just rearranged for different purposes.

Strategy: Treat It Like Solving an Equation

Rearranging is just like solving for a variable in an equation! Use **inverse operations** to isolate the target variable.

Example 1: Linear Formula**Make x the subject of: $y = 3x + 2$** **Step 1:** Subtract 2 from both sides (undo addition)

$$y - 2 = 3x + 2 - 2$$

$$y - 2 = 3x$$

Step 2: Divide by 3 (undo multiplication)

$$\frac{y - 2}{3} = \frac{3x}{3}$$

$$x = \frac{y - 2}{3}$$

Check: If $y = 8$, then $x = \frac{8-2}{3} = 2$. Original: $8 = 3(2) + 2 = 8$ **Example 2: Formula With Multiplication****Make r the subject of: $C = 2\pi r$** This formula relates circumference (C) to radius (r).**Step 1:** Identify what's attached to r

- r is being multiplied by 2π

Step 2: Divide both sides by 2π

$$\frac{C}{2\pi} = \frac{2\pi r}{2\pi}$$

$$r = \frac{C}{2\pi}$$

Meaning: If you know the circumference, divide by 2π to find the radius.

Example 3: Formula With Division

Make h the subject of: $A = \frac{1}{2}bh$

This is the area formula for a triangle.

Step 1: Identify what's with h

- h is being multiplied by $\frac{1}{2}b$

Step 2: Divide both sides by $\frac{1}{2}b$ (or multiply by its reciprocal $\frac{2}{b}$)

$$\frac{A}{\frac{1}{2}b} = h$$

Simplify: $\frac{A}{\frac{1}{2}b} = \frac{2A}{b}$

$$h = \frac{2A}{b}$$

Meaning: If you know area and base, use this to find height.

Check: If $A = 20$ and $b = 4$, then $h = \frac{2(20)}{4} = 10$. Original: $20 = \frac{1}{2}(4)(10) = 20$

Example 4: Formula With Powers (Needs Square Root)

Make a the subject of: $V = \frac{1}{3}a^2h$

This is the volume formula for a pyramid.

Step 1: Multiply both sides by 3 (undo $\frac{1}{3}$)

$$3V = a^2h$$

Step 2: Divide both sides by h (undo multiplication by h)

$$\frac{3V}{h} = a^2$$

Step 3: Take the square root (undo the square)

$$a = \sqrt{\frac{3V}{h}}$$

Or written another way: $a = \sqrt{\frac{3V}{h}} = \frac{\sqrt{3V}}{\sqrt{h}}$

Meaning: If you know volume and height, use this to find the base side a .

Step-by-Step Strategy

- 1. **Identify the target variable** (the one you want to isolate)
- 2. **Work backwards through operations** using inverses
- 3. **Order matters:**
 - Undo addition/subtraction FIRST (loose operations)
 - Then undo multiplication/division
 - Then undo powers/roots
- 4. **Do the same operation to BOTH sides**

Remember

Common mistakes:

- Forgetting to apply the operation to ALL terms
- Forgetting that when you divide by something, divide the ENTIRE other side
- Forgetting that $\sqrt{x^2} = |x|$ (absolute value), but usually we just write x