Linear & Simultaneous Equations

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Linear equations are the foundation for much of CSEC algebra. They express balance: both sides remain equal as the unknown is isolated.

Simultaneous equations extend the same idea to two unknowns. In Paper 02, these questions often appear as word problems, graph intersections, or parts of a larger algebra task. Always state what your variables represent, solve step by step, and check that the solution makes sense in the original situation.

What Is a Linear Equation?

A **linear equation**is an equation where the variable (like x) appears only to the first power. The goal is to find the value of x that makes the equation TRUE.

$$3x + 5 = 14 \leftarrow \text{We want to find what } x \text{ must be}$$

The Fundamental Strategy: Isolate the Variable

The key principle is **"undo operations one at a time."**

If something is being ADDED, subtract it.
If something is being MULTIPLIED, divide it.
If something is being DIVIDED, multiply it.

Always do the SAME operation to both sides of the equals sign.

Order of Operations (Undoing)

When solving, undo operations in REVERSE order of PEMDAS:

- 1. Undo addition/subtraction FIRST (the "loose" operations)
- 2. Then undo multiplication/division (the "attached" operations)
- 3. Then undo powers (if any)

Example 1: Simple Linear Equation

Solve $3x + 5 = 14$:

Step 1: Identify what's happening to x

- x is being multiplied by 3: $3x$
- Then 5 is being added: $3x + 5$

Step 2: Undo addition FIRST (subtract 5 from both sides)

$$3x + 5 - 5 = 14 - 5$$

$$3x = 9$$

Step 3: Undo multiplication (divide both sides by 3)

$$\frac{3x}{3} = \frac{9}{3}$$

$$x = 3$$

Step 4: Check your answer

$$3(3) + 5 = 9 + 5 = 14$$

The answer: $x = 3$

Example 2: Equation With Fractions

Solve $\frac{2x-3}{4} = 5$:

Step 1: What's happening to x?

- $(2x-3)$ is being divided by 4

Step 2: Undo division FIRST (multiply both sides by 4)

$$4 \times \frac{2x-3}{4} = 5 \times 4$$

$$2x - 3 = 20$$

Step 3: Undo subtraction (add 3 to both sides)

$$2x - 3 + 3 = 20 + 3$$

$$2x = 23$$

Step 4: Undo multiplication (divide both sides by 2)

$$x = \frac{23}{2} = 11.5$$

Check: $\frac{2(11.5)-3}{4} = \frac{23-3}{4} = \frac{20}{4} = 5$

Example 3: Brackets on Both Sides

Solve $3(x-2) = 2(x+1)$:

Step 1: Expand both sides

$$3x - 6 = 2x + 2$$

Step 2: Get all x terms on one side. Subtract $2x$ from both sides

$$3x - 2x - 6 = 2x - 2x + 2$$

$$x - 6 = 2$$

Step 3: Undo subtraction (add 6 to both sides)

$$x - 6 + 6 = 2 + 6$$

$$x = 8$$

Check: $3(8-2) = 3(6) = 18$ and $2(8+1) = 2(9) = 18$

Strategy: Get All Variables on One Side

When the variable appears on BOTH sides, move all of it to one side:

📋 **Example**

Solve $5x + 3 = 2x + 12$**:**

Step 1: Move variable terms to LEFT side (subtract $2x$)

$$5x - 2x + 3 = 2x - 2x + 12$$

$$3x + 3 = 12$$

Step 2: Move number terms to RIGHT side (subtract 3)

$$3x = 9$$

Step 3: Solve

$$x = 3$$

🧠 **Remember**

Golden Rule: Whatever you do to one side of the equals sign, MUST do to the other side.

The equation stays balanced, like a seesaw. You can't tilt one side without tilting the other!

Checking Your Solution

ALWAYS substitute your answer back into the ORIGINAL equation to verify.

If it works, you're done.

If it doesn't work, you made an error somewhere.

Simultaneous Linear Equations

Solving Two Equations in Two Unknowns

Method 1: Substitution

📋 **Example**

$$y = x + 3 \quad ...(1)$$

$$2x + y = 9 \quad ...(2)$$

Substitute (1) into (2):

$$2x + (x + 3) = 9$$

$$3x + 3 = 9$$

$$3x = 6$$

$$x = 2$$

From (1): $y = 2 + 3 = 5$

Solution: $(2, 5)$

Check in (2): $2(2) + 5 = 4 + 5 = 9$

Method 2: Elimination

📋 **Example**

$$3x + 2y = 12 \quad ...(1)$$

$$2x - 2y = 3 \quad ...(2)$$

Add equations to eliminate y:

$$(3x + 2y) + (2x - 2y) = 12 + 3$$

$$5x = 15$$

$$x = 3$$

Substitute into (1): $3(3) + 2y = 12 \Rightarrow 9 + 2y = 12 \Rightarrow y = 1.5$

Solution: $(3, 1.5)$

Method 3: Graphical Method

Plot both equations and find the intersection point.