Composite & Inverse Functions

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Composite and inverse functions are about processes. A composite function performs one rule and then another; an inverse function reverses a rule and takes you back to the original input.

In CSEC, this topic sits in Relations, Functions and Graphs, where questions may ask you to evaluate, form, or rearrange functions. Pay close attention to order: $f(g(x))$ usually gives a different result from $g(f(x))$. For inverse functions, explain how each algebraic step undoes the original operation.

What Is a Composite Function?

A **composite function** is when you apply one function, then apply another function to the result.

Notation: $(f \circ g)(x)$ or $f(g(x))$

Read as: "f composed with g of x"

Process:

- 1. First, find $g(x)$ (apply inner function)
- 2. Then, apply f to that result: $f(g(x))$

Important: $f(g(x)) \neq g(f(x))$ (order matters!)

Finding Composite Functions

📋 **Example**

Given $f(x) = 2x + 1$ **and** $g(x) = x^2$**, find** $f(g(x))$**:**

Step 1: Identify what we're doing

$$f(g(x)) = \text{apply } g \text{ first, then apply } f$$

Step 2: Find $g(x) = x^2$

Step 3: Apply f to the result: wherever we see x in the definition of f, substitute $g(x) = x^2$

$$f(g(x)) = f(x^2) = 2(x^2) + 1 = 2x^2 + 1$$

Answer: $f(g(x)) = 2x^2 + 1$

This is a new function that takes x, squares it, multiplies by 2, and adds 1.

📋 **Example**

Given $f(x) = 2x + 1$ **and** $g(x) = x^2$**, find** $g(f(x))$**:**

Step 1: Apply f first to get $f(x) = 2x + 1$

Step 2: Apply g to the result: wherever x appears in g, substitute $f(x) = 2x + 1$

$$g(f(x)) = g(2x + 1) = (2x + 1)^2$$

Step 3: Expand

$$g(f(x)) = 4x^2 + 4x + 1$$

Compare:

- $f(g(x)) = 2x^2 + 1$
- $g(f(x)) = 4x^2 + 4x + 1$

They're DIFFERENT! Order matters!

Evaluating Composite Functions

To find $f(g(3))$ where $f(x) = 2x + 1$ and $g(x) = x^2$:

Method 1: Find $g(x)$ **first, then** f **of that**

$$g(3) = 3^2 = 9$$

$$f(g(3)) = f(9) = 2(9) + 1 = 19$$

Method 2: Use the composite function formula we found

$$f(g(x)) = 2x^2 + 1$$

$$f(g(3)) = 2(3)^2 + 1 = 18 + 1 = 19$$

Both methods give the same answer (as they should!).

Inverse Functions

What Is an Inverse Function?

An **inverse function** "undoes" what the original function does.

If f takes input x to output y, then f^{-1} takes input y back to output x.

Notation: $f^{-1}(x)$ (read as " f inverse of x ")

Key property:

$$f(f^{-1}(x)) = x$$

$$f^{-1}(f(x)) = x$$

When Does an Inverse Exist?

Not every function has an inverse! A function has an inverse ONLY if it's **one-to-one** (injective):

- Each output comes from exactly one input
- No two different inputs produce the same output
- Passes the **horizontal line test:** any horizontal line crosses the graph at most once

📋 **Example**

Does $f(x) = 2x + 3$ have an inverse?

Check: Different inputs give different outputs

- $f(1) = 5$
- $f(2) = 7$
- $f(3) = 9$
- ...

Each input x produces a unique output. YES, it has an inverse.

Does $g(x) = x^2$ have an inverse (for all real numbers)?

Check: Multiple inputs give same output

- $g(2) = 4$
- $g(-2) = 4$

Same output (4) from different inputs (2 and -2). NO, it doesn't have an inverse for all reals.

(Note: If we restrict the domain to $x \geq 0$, then g is one-to-one and has an inverse.)

Finding Inverse Functions Algebraically

To find $f^{-1}(x)$ from $f(x)$:

Step 1: Write $y = f(x)$

Step 2: Swap x and y

Step 3: Solve for y in terms of x

Step 4: Replace y with $f^{-1}(x)$

📋 **Example**

Find the inverse of $f(x) = 2x + 3$**:**

Step 1: Write $y = 2x + 3$

Step 2: Swap x and y

$$x = 2y + 3$$

Step 3: Solve for y

$$x - 3 = 2y$$

$$y = \frac{x - 3}{2}$$

Step 4: Write as inverse function

$$f^{-1}(x) = \frac{x - 3}{2}$$

Verify:

- $f(f^{-1}(x)) = f\left(\frac{x-3}{2}\right) = 2 \cdot \frac{x-3}{2} + 3 = (x - 3) + 3 = x$
- $f^{-1}(f(x)) = f^{-1}(2x + 3) = \frac{(2x+3)-3}{2} = \frac{2x}{2} = x$

📋 **Example**

Find the inverse of $f(x) = x^3 - 2$**:**

Step 1: Write $y = x^3 - 2$

Step 2: Swap x and y

$$x = y^3 - 2$$

Step 3: Solve for y

$$x + 2 = y^3$$

$$y = \sqrt[3]{x + 2}$$

Step 4: Write as inverse

$$f^{-1}(x) = \sqrt[3]{x + 2}$$

Verify: $f(f^{-1}(x)) = (\sqrt[3]{x + 2})^3 - 2 = (x + 2) - 2 = x$

Inverse Functions and Composition

The fundamental relationship:

$$f \circ f^{-1} = I \text{ (identity function)}$$

$$f^{-1} \circ f = I$$

And if you have composite functions:

$$(f \circ g)^{-1} = g^{-1} \circ f^{-1}$$

Note the **order reverses** when taking inverses of composites!

📋 **Example**

If $f(x) = 2x + 1$ and $g(x) = x - 3$, find $(f \circ g)^{-1}$:

Method 1: Find $(f \circ g)$ first, then invert

$$f(g(x)) = f(x-3) = 2(x-3) + 1 = 2x - 5$$

Now invert $y = 2x - 5$:

$$x = 2y - 5$$

$$y = \frac{x+5}{2}$$

$$(f \circ g)^{-1}(x) = \frac{x+5}{2}$$

Method 2: Use $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$

Find inverses:

- $f^{-1}(x) = \frac{x-1}{2}$
- $g^{-1}(x) = x + 3$

$$g^{-1}(f^{-1}(x)) = g^{-1}\left(\frac{x-1}{2}\right) = \frac{x-1}{2} + 3 = \frac{x-1+6}{2} = \frac{x+5}{2}$$

Both methods agree!