Mathematics Formula Book

Matthew Williams • Math • May 21, 2026

Number Theory and Computation

Place value (base 10):each digit position represents a power of 10. In base b, each position represents a power of b and digits must be less than b.

Standard form (scientific notation): $a \times 10^n$ where $1 \leq a < 10$ and n is an integer.

Rounding: round up if the next digit is 5 or more, otherwise round down. Report significant figures from the first non-zero digit.

Surds: $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$, $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$, $\sqrt{a} \times \sqrt{a} = a$.

Rationalise the denominator using the conjugate: $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$.

Number types:natural numbers $\mathbb{N}$ (1, 2, 3, ...), integers $\mathbb{Z}$, rationals $\mathbb{Q}$ (expressible as $\frac{p}{q}$, $q \neq 0$), irrationals (non-terminating, non-repeating), reals $\mathbb{R}$.

HCF and LCM via prime factorisation:

- HCF: take each prime to its **lowest** power in either factorisation.
- LCM: take each prime to its **highest** power in either factorisation.
- Relationship: $\mathrm{HCF}(a, b) \times \mathrm{LCM}(a, b) = a \times b$.

Arithmetic sequences:

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d]$$

where a is the first term and d is the common difference.

Consumer Arithmetic

Profit and loss:

$$\text{Profit/Loss} = \text{Selling Price} - \text{Cost Price}$$

$$\text{Profit \%} = \frac{\text{Profit}}{\text{Cost Price}} \times 100\%$$

A **markup** is a percentage added to the cost price; a **markdown/discount** is a percentage reduction from the marked price.

Percentage change:

$$\text{\% change} = \frac{\text{new} - \text{old}}{\text{old}} \times 100\%$$

Simple interest:

$$I = \frac{PRT}{100} \qquad A = P + I = P\left(1 + \frac{RT}{100}\right)$$

where P is the principal, R is the annual rate (%), and T is the time in years.

Compound interest:

$$A = P\left(1 + \frac{R}{100}\right)^n$$

where n is the number of compounding periods. Compound interest $= A - P$.

Appreciation and depreciation use the same compound formula:

$$\text{New value} = \text{Original} \times \left(1 \pm \frac{R}{100}\right)^n$$

Use $+$ for appreciation, $-$ for depreciation.

Hire purchase:

deposit paid upfront, then equal instalments. Total HP cost = deposit + (instalment × number of instalments). The HP price is usually more than the cash price.

Currency conversion: multiply by the exchange rate to convert from the base currency; divide to reverse. Always label the currency at each step.

Tax: Value Added Tax (VAT) and income tax are percentages of the taxable amount. Net pay = gross pay minus deductions.

Sets

Notation:

Symbol	Meaning
[Math: \in]	is an element of
[Math: \notin]	is not an element of
[Math: \subseteq]	is a subset of
[Math: \cup]	union (or)
[Math: \cap]	intersection (and)
[Math: A']	complement of [Math: A]
[Math: n(A)]	number of elements in [Math: A]
[Math: \varnothing] or [Math: \{\}]	empty set
[Math: U]	universal set

Key results:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

$$A \cup A' = U \qquad A \cap A' = \varnothing \qquad (A')' = A$$

De Morgan's laws:

$$(A \cup B)' = A' \cap B' \qquad (A \cap B)' = A' \cup B'$$

A **Venn diagram** shows sets as overlapping circles inside a rectangle (the universal set). Fill regions starting from the innermost intersection.

Algebra

Expanding brackets: $(a+b)(c+d) = ac + ad + bc + bd$

Special products:

$$a^2 - b^2 = (a+b)(a-b)$$

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$

Laws of indices (same base only):

Law	Rule
[Math: a^m \times a^n]	[Math: a^{m+n}]
[Math: a^m \div a^n]	[Math: a^{m-n}]
[Math: (a^m)^n]	[Math: a^{mn}]
[Math: (ab)^n]	[Math: a^n b^n]
[Math: a^0]	[Math: 1]
[Math: a^{-n}]	[Math: \dfrac{1}{a^n}]

Solving linear equations: isolate the variable by performing the same inverse operation on both sides.

Simultaneous linear equations (two unknowns):

- **Substitution:** rearrange one equation, substitute into the other.
- **Elimination:** multiply equations so one variable cancels when added or subtracted.

Quadratic formula:for $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant $\Delta = b^2 - 4ac$:

[Math: \Delta]	Nature of roots
[Math: \Delta > 0]	Two distinct real roots
[Math: \Delta = 0]	One repeated real root
[Math: \Delta < 0]	No real roots

Vertex (turning point) of $y = ax^2 + bx + c$:

$$x_v = -\frac{b}{2a} \qquad y_v = c - \frac{b^2}{4a}$$

Parabola opens upward if $a > 0$ (minimum), downward if $a < 0$ (maximum).

Direct variation: $y = kx$, so $\frac{y}{x} = k$ (constant). Graph is a straight line through the origin.

Inverse variation: $y = \frac{k}{x}$, so $xy = k$ (constant). Graph is a hyperbola.

Changing the subject: apply inverse operations in reverse PEMDAS order to isolate the required variable.

Solving inequalities: use the same method as equations, but **reverse the inequality sign** when multiplying or dividing both sides by a negative number.

Number line conventions:

- Open circle ○ : endpoint not included ($<$ or $>$).
- Filled circle ● : endpoint included ($\leq$ or $\geq$).

Relations, Functions, and Graphs

Function notation: $f(x)$ means the output of function f for input x. Evaluate by substituting the input for every occurrence of the variable.

Domain: the set of permitted inputs. **Range (image):** the set of actual outputs.

Vertical line test: a graph represents a function if and only if every vertical line crosses it at most once.

Composite functions: $(f \circ g)(x) = f(g(x))$: g is applied first, then f. In general, $f \circ g \neq g \circ f$.

Inverse function f^{-1} : swap x and y, then solve for y.

Verification: $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

Linear function $y = mx + c$:

- Gradient $m = \frac{y2 - y1}{x2 - x1} = \frac{\text{rise}}{\text{run}}$
- y -intercept $= c$ (where the line crosses the y -axis)
- x -intercept: set $y = 0$ and solve

Parallel lineshave equal gradients. **Perpendicular lines**satisfy $m1m2 = -1$.

Equation of a linethrough $(x1, y1)$ with gradient m:

$$y - y_1 = m(x - x_1)$$

Midpointof $(x1, y1)$ and $(x2, y2)$:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Distance between two points:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Graphical solution of simultaneous equations: plot both lines; the solution is their intersection point.

Linear inequalitieson a graph: use a dashed boundary line for strict inequalities ($<$, $>$), a solid line for $\leq$ or $\geq$. The feasible region is the set of points satisfying all constraints. The optimum of an objective function occurs at a vertex of the feasible region.

Geometry

Angle facts:

Rule	Statement
Angles on a straight line	sum to [Math: 180°]
Angles at a point	sum to [Math: 360°]
Vertically opposite angles	equal
Corresponding angles (parallel lines)	equal
Alternate angles (parallel lines)	equal
Co-interior / allied angles (parallel lines)	sum to [Math: 180°]

Triangle angles:interior angles sum to 180°

; an exterior angle equals the sum of the two non-adjacent interior angles.

Polygon angle sumsfor an n-sided polygon:

- Interior angle sum: $(n-2)\times 180^\circ$
- Each interior angle (regular): $\frac{(n-2)\times 180^\circ}{n}$
- Each exterior angle (regular): $\frac{360^\circ}{n}$

Circle theorems:

Theorem	Statement
Angle in semicircle	[Math: = 90°]
Angle at centre	[Math: = 2 \times] angle at circumference (same arc)
Angles in same segment	equal
Cyclic quadrilateral	opposite angles sum to [Math: 180°]
Tangent to radius	perpendicular at point of contact
Tangents from external point	equal in length
Alternate segment theorem	angle between tangent and chord [Math: =] angle in alternate segment

Congruence conditions (triangles): SSS, SAS, ASA (or AAS), RHS.

Similarity:

corresponding angles equal; corresponding sides in the same ratio. If scale factor is k:

- Lengths scale by k
- Areas scale by k^2
- Volumes scale by k^3

Transformations:

Transformation	Description
Translation by vector [Math: (a, b)]	every point moves [Math: a] right, [Math: b] up
Reflection in [Math: y = x]	swap [Math: x] and [Math: y] coordinates
Rotation [Math: \theta°] anticlockwise about origin	multiply by rotation matrix (see below)
Enlargement, centre [Math: O], factor [Math: k]	distance from [Math: O] multiplied by [Math: k]

Rotation matrix for $\theta°$ anticlockwise about the origin:

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

Measurement

Perimeter and area:

Shape	Perimeter	Area
Rectangle	[Math: P = 2(l + w)]	[Math: lw]
Triangle	[Math: a + b + c]	[Math: \frac{1}{2}bh]
Parallelogram	[Math: P = 2(a + b)]	[Math: bh]
Trapezium	[Math: a + b + c + d]	[Math: \frac{1}{2}(a + b)h]
Circle	[Math: C = 2\pi r]	[Math: \pi r^2]

Circle arc and sector(angle θ in degrees):

$$\text{Arc length} = \frac{\theta}{360} \times 2\pi r \qquad \text{Sector area} = \frac{\theta}{360} \times \pi r^2$$

Surface area and volume:

Shape	Surface area	Volume
Cuboid	[Math: SA = 2(lw + lh + wh)]	[Math: lwh]
Cylinder	[Math: SA = 2\pi r^2 + 2\pi rh]	[Math: \pi r^2 h]
Cone	[Math: \pi r^2 + \pi rl] ([Math: l] = slant height)	[Math: \frac{1}{3}\pi r^2 h]
Sphere	[Math: SA = 4\pi r^2]	[Math: \frac{4}{3}\pi r^3]
Pyramid	base area + lateral faces	[Math: \frac{1}{3} \times \text{base area} \times h]

Speed, distance, time:

$$D = ST \qquad S = \frac{D}{T} \qquad T = \frac{D}{S}$$

Convert km/h to m/s by multiplying by $\frac{5}{18}$; convert m/s to km/h by multiplying by $\frac{18}{5}$.

Scale drawings: $\text{actual length} = \text{map length} \times \text{scale factor}$

Density and pressure:

$$\rho = \frac{m}{V} \qquad P = \frac{F}{A}$$

Unit conversions (area and volume):when converting lengths by a factor of k, areas scale by k^2 and volumes scale by k^3.

Margin of error:when a measurement is rounded to the nearest unit u, the true value lies within half a unit above or below: $\text{measured value} \pm \frac{u}{2}$.

Trigonometry

SOHCAHTOA (right-angled triangles only):

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

Pythagoras' theorem:

$$a^2 + b^2 = c^2 \qquad c^2 - a^2 = b^2$$

Exact values:

[Math: \theta]	30°	45°	60°
[Math: \sin\theta]	[Math: \dfrac{1}{2}]	[Math: \dfrac{\sqrt{2}}{2}]	[Math: \dfrac{\sqrt{3}}{2}]
[Math: \cos\theta]	[Math: \dfrac{\sqrt{3}}{2}]	[Math: \dfrac{\sqrt{2}}{2}]	[Math: \dfrac{1}{2}]
[Math: \tan\theta]	[Math: \dfrac{1}{\sqrt{3}}]	1	[Math: \sqrt{3}]

Angles of elevation and depression:

both measured from the horizontal. The angle of elevation from A to B equals the angle of depression from B to A.

Sine rule(any triangle, labelled with sides a, b, c opposite angles A, B, C):

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Use when given: two angles and one side (AAS), or two sides and a non-included angle (SSA, watch for ambiguous case).

Cosine rule:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Use when given: two sides and the included angle (SAS), or all three sides (SSS).

Area of triangle:

$$\text{Area} = \frac{1}{2}ab\sin C$$

Bearings: measured clockwise from North, always written as three digits (e.g., 045°, 270°). Draw a North line at every point in a bearing problem.

Vectors

Column vector notation:

$$\vec{v} = \begin{pmatrix} x \\ y \end{pmatrix} \qquad \text{or} \qquad \vec{v} = x\mathbf{i} + y\mathbf{j}$$

Magnitude:

$$|\vec{v}| = \sqrt{x^2 + y^2}$$

Addition and subtraction:

$$\begin{pmatrix} a \\ b \end{pmatrix} \pm \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a \pm c \\ b \pm d \end{pmatrix}$$

Scalar multiplication:multiply every component by the scalar. Multiplying by k scales the magnitude by $|k|$ and reverses direction if $k < 0$.

Position vectorof point P: $\overrightarrow{OP} = \mathbf{p}$ (from origin O to P).

Displacement vector: $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$

Parallel vectors: $\vec{u}$ and $\vec{v}$ are parallel if $\vec{u} = k\vec{v}$ for some scalar k.

Collinear points: A, B, C are collinear if $\overrightarrow{AC} = k\overrightarrow{AB}$.

Matrices

Matrix dimensions: an $m \times n$ matrix has m rows and n columns.

Addition/subtraction: add or subtract corresponding entries; only defined for matrices of the same dimensions.

Scalar multiplication: multiply every entry by the scalar.

Matrix multiplication: AB is defined only if the number of columns of A equals the number of rows of B. The (i, j) entry of AB is the dot product of row i of A with column j of B.

For 2×2 matrices A and B:

$$AB = \begin{pmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{pmatrix}$$

Note: in general, $AB \neq BA$.

Determinant of a 2×2 matrix:

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

An inverse exists if and only if $\det \neq 0$; the matrix is then called non-singular.

Inverse of a 2×2 matrix:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Solving simultaneous equations with matrices: for $A\mathbf{x} = \mathbf{b}$, the solution is $\mathbf{x} = A^{-1}\mathbf{b}$ (provided A^{-1} exists).

Common transformation matrices (applied by multiplying from the left):

Reflection in the x-axis:

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Reflection in the y-axis:

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Reflection in $y = x$:

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Rotation 90° anticlockwise about O:

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

Rotation 180° about O:

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

Enlargement factor k, centre O:

$$\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$$

Statistics and Probability

Mean (ungrouped data):

$$\bar{x} = \frac{\sum x}{n}$$

Mean (grouped/frequency data):

$$\bar{x} = \frac{\sum fx}{\sum f}$$

where x is the midpoint of each class and f is the frequency.

Median:the middle value when data are ordered. For n values, the median is at position $\frac{n+1}{2}$.

Mode: the value (or class) with the highest frequency.

Range: $\mathbf{maximum - minimum}$

Interquartile range (IQR): $Q3 - Q1$

Semi-interquartile range: $\frac{Q3 - Q1}{2}$

Cumulative frequency

(ogive): plot cumulative frequency against the upper class boundary. Read $Q1$ at $\frac{n}{4}$, median at $\frac{n}{2}$, and $Q3$ at $\frac{3n}{4}$.

Pie chart sector angle:

$$\text{Angle} = \frac{\text{frequency}}{\text{total}} \times 360^\circ$$

Theoretical probability:

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of equally likely outcomes}}$$

Complement: $P(E') = 1 - P(E)$

Mutually exclusive events: $P(A \cup B) = P(A) + P(B)$

Independent events: $P(A \cap B) = P(A) \times P(B)$

Tree diagrams: multiply probabilities along branches; add probabilities across branches for the same outcome. The probabilities on branches from any node must sum to 1.

Experimental probability:

$$P(E) \approx \frac{\text{number of times event occurred}}{\text{total number of trials}}$$

As the number of trials increases, experimental probability approaches theoretical probability.