

# Congruence, Similarity & Transformations

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Congruence, similarity, and transformations are about how shapes relate to each other. Some shapes stay exactly the same size, some are scaled versions of each other, and some are moved around the plane.

In CSEC Geometry, you may need to identify a transformation, give its properties, or justify why two figures are congruent or similar. Use precise language: translation vector, line of reflection, centre and angle of rotation, scale factor, or corresponding sides. Clear descriptions earn marks.

## Congruent Figures

Congruent figures can be moved, turned, or flipped to match exactly. Because size does not change, corresponding sides and corresponding angles are equal.

**Congruent** means identical in shape and size. One can be transformed into the other.

Triangles are congruent if:

- **SSS** (Side-Side-Side): All three sides equal
- **SAS** (Side-Angle-Side): Two sides and included angle equal
- **ASA** (Angle-Side-Angle): Two angles and included side equal
- **RHS** (Right angle-Hypotenuse-Side): Right triangle with hypotenuse and one side equal

## Similar Figures

Similar figures have the same shape but not necessarily the same size. Angles stay equal, while side lengths are multiplied by a scale factor.

**Similar** means same shape but different size.

Triangles are similar if:

- **AAA** (Angle-Angle-Angle): All angles equal (only need 2 angles since 3rd is determined)
- **SSS** (Side-Side-Side): All sides proportional
- **SAS** (Side-Angle-Side): Two sides proportional with included angle equal

**Key property:** Corresponding angles are equal and corresponding sides are proportional.

$$\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'} = k \text{ (scale factor)}$$

# Transformations

## Translation (Sliding)

A translation moves every point the same distance in the same direction. The shape does not turn, flip, or resize.

Move every point the same distance in the same direction.

**Representation:** Column vector  $\begin{pmatrix} x \\ y \end{pmatrix}$  shows displacement

**Example:** Translate point  $(2, 3)$  by  $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

New position:  $(2 + 3, 3 - 2) = (5, 1)$

## Reflection (Flipping)

A reflection creates a mirror image. Every point and its image are the same perpendicular distance from the mirror line.

Flip over a line (mirror line). Every point and its image are equidistant from the mirror line.

**Common mirror lines:**

- $x$  -axis:  $(x, y) \rightarrow (x, -y)$
- $y$  -axis:  $(x, y) \rightarrow (-x, y)$
- Line  $y = x$  :  $(x, y) \rightarrow (y, x)$

## Rotation (Turning)

A rotation needs three pieces of information: center, angle, and direction. Missing any one of these makes the transformation description incomplete.

Turn around a fixed point (center) by a certain angle, clockwise or counterclockwise.

**Key information needed:**

- Center of rotation
- Angle of rotation
- Direction (clockwise or counterclockwise)

**Common rotations:**  $90^\circ$ ,  $180^\circ$ ,  $270^\circ$

## Enlargement/Reduction (Scaling)

Enlargement changes size but preserves shape. Distances from the center of enlargement are multiplied by the scale factor.

Change size by a scale factor from a center point.

**Scale factor  $k$ :**

- If  $k > 1$  : Enlargement (gets bigger)
- If  $0 < k < 1$  : Reduction (gets smaller)
- If  $k < 0$  : Enlargement with  $180^\circ$  rotation

**For point  $(x, y)$  from center  $(0, 0)$ :**

$$\text{Image} = (kx, ky)$$

## Combinations of Transformations

When transformations are combined, order can matter. Perform them one at a time and use the image from the first transformation as the object for the next.

Transformations can be combined (apply one, then another).

**Example:** Translate then reflect produces same result regardless of order? NO! Order matters.