

# Pythagoras' Theorem & Trigonometry

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Right-angled triangle questions come down to matching available information to the correct relationship. Side lengths alone call for Pythagoras; angles combined with sides call for trigonometry.

CSEC often tests this topic in practical contexts such as ladders, heights, ramps, shadows, and distances. Before calculating, mark the hypotenuse, opposite side, and adjacent side on the diagram. That simple labelling step turns a word problem into a method you can explain.

**In any right-angled triangle:**

$$a^2 + b^2 = c^2$$

Where  $c$  is the hypotenuse (longest side, opposite the right angle) and  $a$ ,  $b$  are the other sides.

## Finding the Hypotenuse

### Example

Triangle with sides 3 and 4. Find hypotenuse:

$$c^2 = 3^2 + 4^2 = 9 + 16 = 25$$

$$c = 5$$

## Finding a Missing Side

### Example

Right triangle with hypotenuse 13 and one side 5. Find other side:

$$5^2 + b^2 = 13^2$$

$$25 + b^2 = 169$$

$$b^2 = 144$$

$$b = 12$$

## Application: Distance Formula

Distance between  $(x_1, y_1)$  and  $(x_2, y_2)$ :

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

This IS Pythagoras' theorem!

## Trigonometric Ratios

### Right Triangle Basics

In a right triangle with acute angle  $\theta$ :

- **Opposite:** Side opposite to angle  $\theta$
- **Adjacent:** Side next to angle  $\theta$  (not hypotenuse)
- **Hypotenuse:** Longest side (opposite the right angle)

### The Three Ratios

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

### Memory aid: SOH-CAH-TOA

- Sine = Opposite/Hypotenuse
- Cosine = Adjacent/Hypotenuse
- Tangent = Opposite/Adjacent

## Finding Ratios from Angles

Use a calculator:

- $\sin(30^\circ) = 0.5$
- $\cos(30^\circ) = 0.866\dots$
- $\tan(30^\circ) = 0.577\dots$
  
- $\sin(45^\circ) = 0.707\dots$
- $\cos(45^\circ) = 0.707\dots$
- $\tan(45^\circ) = 1$
  
- $\sin(60^\circ) = 0.866\dots$
- $\cos(60^\circ) = 0.5$
- $\tan(60^\circ) = 1.732\dots$

## Solving Right Triangles

To find unknown side:

### Example

Right triangle: hypotenuse = 10, angle =  $35^\circ$ . Find opposite side:

$$\sin(35^\circ) = \frac{\text{opposite}}{10}$$

$$\text{opposite} = 10 \times \sin(35^\circ) = 10 \times 0.574 = 5.74$$

To find unknown angle:

### Example

Right triangle: opposite = 7, hypotenuse = 10. Find angle:

$$\sin(\theta) = \frac{7}{10} = 0.7$$

$$\theta = \sin^{-1}(0.7) = 44.4^\circ$$