Sine Rule, Cosine Rule & Bearings

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Not every triangle has a right angle, so SOH-CAH-TOA is not always enough. The sine rule and cosine rule let you solve non-right-angled triangles by matching known sides and angles.

CSEC often uses these ideas in bearings, navigation, and survey-style word problems. Draw a neat diagram first, mark north lines for bearings, and label opposite side-angle pairs. The written setup is important because it shows why the sine rule or cosine rule is appropriate.

These rules work for ANY triangle (not just right triangles).

Sine Rule

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

Where lowercase letters are sides and uppercase letters are opposite angles.

Use when you know:

- Two angles and one side (find other sides)
- Two sides and an angle opposite one of them (find other angles)

<JSXGraph id="geom-sine-rule" title="Sine rule: a/sin A = b/sin B = c/sin C" boundingbox={[-1, 5, 7, -1]} axis={false} height={280} code="board.create('polygon',[[0,0],[6,0],[2,4]],{borders:{strokeColor:'#1d4ed8',strokeWidth:2},fillColor:c.fill
/>

📋 **Example**

Triangle: side [Math: a = 10], angle [Math: A = 30°], angle [Math: B = 50°]. Find side [Math: b]:

[MathBlock]
\frac{10}{\sin(30°)} = \frac{b}{\sin(50°)}
[/MathBlock]

[MathBlock]
\frac{10}{0.5} = \frac{b}{0.766}
[/MathBlock]

[MathBlock]
20 = \frac{b}{0.766}
[/MathBlock]

[MathBlock]
b = 20 \times 0.766 = 15.3
[/MathBlock]

Cosine Rule

c^2 = a^2 + b^2 - 2ab\cos(C)

Or rearranged to find angles:

\cos(C) = \frac{a^2 + b^2 - c^2}{2ab}

Use when you know:

- All three sides (find angles)
- Two sides and included angle (find third side)

<JSXGraph id="geom-cos-rule" title="Cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$" boundingbox={[-1, 5, 7, -1]} axis={false} height={280} code="board.create('polygon',[[0,0],[6,0],[2,4]],{borders:{strokeColor:'#1d4ed8',strokeWidth:2},fillColor:c.fill
C'],{fontSize:13,color:'#dc2626'});" />

📋 **Example**

Triangle: sides [Math: a = 5], [Math: b = 7], angle [Math: C = 60°]. Find side [Math: c]:

[MathBlock]
c^2 = 5^2 + 7^2 - 2(5)(7)\cos(60°)
[/MathBlock]

[MathBlock]
c^2 = 25 + 49 - 70(0.5)
[/MathBlock]

[MathBlock]
c^2 = 74 - 35 = 39
[/MathBlock]

[MathBlock]
c = \sqrt{39} = 6.24
[/MathBlock]

Part 10: Bearings and Navigation

What Is a Bearing?

A **bearing** is a direction, measured as an angle clockwise from North.

- North = 0° (or 360°)
- East = 90°
- South = 180°
- West = 270°

Notation: Written as three digits, e.g., 045°, 127°, 310°

Reading and Writing Bearings

📋 **Example**

A point is at bearing 120° from point A:

- Start facing North from A
- Turn clockwise 120°
- You're now facing Southeast

A point is at bearing 280° from B:

- Start facing North from B
- Turn clockwise 280°
- You're now facing West-Northwest

Finding Bearings Between Points

The bearing from point A to point B is the clockwise angle from North at A to the direction of B.

<JSXGraph id="geom-bearing" title="Bearing measured clockwise from North" boundingbox={[-5, 5, 5, -5]} axis={false} height={320} code="board.create('arrow',[[0,0],[0,4]],{strokeColor:'#16a34a',strokeWidth:2});board.create('text',[0.2,4.2,'N
1.5Math.sin(t);},function(t){return
1.5Math.cos(t);},0,Math.atan2(3,2)],{strokeColor:'#1d4ed8',strokeWidth:2});board.create('text',[0.6,1.7,'056°'
/>

Solving Bearing Problems

Most involve:

- 1. Identifying the triangle formed
- 2. Using known distances and angles
- 3. Applying sine/cosine rules
- 4. Finding bearings or distances

📋 **Example**

Ship travels from port A on bearing 050° for 40 km to reach point B. Then from B on bearing 140° for 30 km to reach point C. Find distance from A to C:

Step 1: Identify the angle at B in the triangle

- At B, one direction is back along bearing 050° + 180° = 230°
- Other direction is bearing 140°
- Angle at B = 230° - 140° = 90°

Step 2: Use Pythagoras (it's a right angle!)

[MathBlock]
AC^2 = 40^2 + 30^2 = 1600 + 900 = 2500
[/MathBlock]

[MathBlock]
AC = 50 \text{ km}
[/MathBlock]

Step 3: Find bearing from A to C
Using trigonometry: [Math: \tan(\angle) = \frac{30}{40} = 0.75]
Bearing ≈050° + small adjustment ≈087°

Part 11: Heights and Distances

Angle of Elevation

The angle you look UP from horizontal to see something above you.

Angle of Depression

The angle you look DOWN from horizontal to see something below you.

Key relationship: Angle of depression from one point = Angle of elevation from other point

Solving Height Problems

📋 **Example**

From ground, looking at top of 30m building at angle of elevation 25°. How far away is the building?

[MathBlock]
\tan(25°) = \frac{30}{\text{distance}}
[/MathBlock]

[MathBlock]
\text{distance} = \frac{30}{\tan(25°)} = \frac{30}{0.466} = 64.4 \text{ m}
[/MathBlock]

📋 **Example**

From cliff top 100m high, looking down at boat at angle of depression 20°. How far from cliff is boat?

Angle of depression 20° = Angle of elevation 20° from boat's perspective

[MathBlock]
\tan(20°) = \frac{100}{\text{distance}}
[/MathBlock]

[MathBlock]
\text{distance} = \frac{100}{\tan(20°)} = \frac{100}{0.364} = 274.6 \text{ m}
[/MathBlock]