

Linear Functions & Graphs

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Linear functions connect algebra to visual patterns. The equation tells you how y changes as x changes, and the graph shows that relationship as a straight line.

In CSEC, graph questions may ask for gradient, intercepts, equations of lines, parallel or perpendicular lines, or graphical solutions to simultaneous equations. Do not only plot points; explain what the gradient and intercept mean. That helps with comprehension and reasoning marks.

What Is a Linear Function?

A **linear function** is a function where the graph is a straight line.

General form:

$$f(x) = mx + c$$

Or:

$$y = mx + c$$

Where:

- m = **slope** (gradient) — how steep the line is
- c = **y-intercept** — where the line crosses the y-axis
- x = input (domain variable)
- y or $f(x)$ = output (range variable)

Understanding Slope (Gradient)

Slope measures how much y changes when x increases by 1.

$$m = \text{slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Interpretation:

- $m > 0$: Line goes UP from left to right (positive slope)
- $m < 0$: Line goes DOWN from left to right (negative slope)

- [Math: $m = 0$]: Horizontal line (flat, no change)
- [Math: m] undefined: Vertical line

Examples of Different Slopes

```
<JSXGraph id="rel-slopes" title="Different slopes: y=2x (steep), y=x, y=0.5x (shallow)"
boundingbox={[-5, 6, 5, -6]} height={340}
code="board.create('functiongraph',[function(x){return
2x;},-3,3},{strokeColor:'#dc2626',strokeWidth:2,name:'y=2x',withLabel:true});board.create('functiongraph',[f
x;},-5,5},{strokeColor:'#2563eb',strokeWidth:2,name:'y=x',withLabel:true});board.create('functiongraph',[fun
0.5x;},-5,5},{strokeColor:'#16a34a',strokeWidth:2,name:'y=0.5x',withLabel:true});" />
```

Forms of Linear Equations

Form 1: Slope-Intercept Form (Most Useful)

$$y = mx + c$$

- Easy to identify: slope is [Math: m], y-intercept is [Math: c]
- Easy to graph: plot [Math: $(0, c)$], then use slope to find more points

Example

Graph [Math: $y = 2x - 3$]:

- **Slope:** [Math: $m = 2$] (go up 2 for every right 1)
- **Y-intercept:** [Math: $c = -3$] (crosses y-axis at [Math: $(0, -3)$])

Plot key points:

- Start at [Math: $(0, -3)$]
- Slope 2 means: right 1, up 2 'next point [Math: $(1, -1)$]
- Continue: [Math: $(2, 1)$], [Math: $(3, 3)$], etc.

```
<JSXGraph id="rel-y2xm3" title="y = 2x - 3" boundingbox={[-2, 4, 5, -5]} height={320}
code="board.create('functiongraph',[function(x){return
2*x-3;},-2,5},{strokeColor:'#2563eb',strokeWidth:2});board.create('point',[0,-3],{name:'(0,-3)',size:3,fillColor:c.re
/>
```

Form 2: Point-Slope Form

$$y - y_1 = m(x - x_1)$$

Use this when you know:

- The slope [Math: m]
- One point [Math: (x_1, y_1)] on the line

Example

Find the equation of a line with slope 3 passing through [Math: $(2, 5)$]:

Step 1: Use point-slope form

$$y - 5 = 3(x - 2)$$

Step 2: Expand

$$y - 5 = 3x - 6$$

Step 3: Rearrange to slope-intercept form

$$y = 3x - 6 + 5$$

$$y = 3x - 1$$

Form 3: Two-Point Form

When you know two points [Math: (x_1, y_1)] and [Math: (x_2, y_2)]:

Step 1: Find slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Step 2: Use point-slope form with either point

Example

Find the equation of a line through $(1, 3)$ and $(4, 12)$:

Step 1: Find slope

$$m = \frac{12 - 3}{4 - 1} = \frac{9}{3} = 3$$

Step 2: Use point-slope with first point $(1, 3)$

$$y - 3 = 3(x - 1)$$

$$y = 3x - 3 + 3$$

$$y = 3x$$

Verify with second point: $y = 3(4) = 12$

Form 4: Standard Form

$$Ax + By + C = 0$$

Or:

$$Ax + By = C$$

Where A , B , C are integers with no common factors.

Convert from slope-intercept to standard:

$$y = 2x - 3 \rightarrow 2x - y - 3 = 0$$

Finding Intercepts

Y-Intercept

The y-intercept is where the line crosses the y-axis (when $x = 0$).

To find: Set $x = 0$ and solve for y .

Example

Find y-intercept of $2x + 3y = 6$:

Set $x = 0$:

$$2(0) + 3y = 6$$

$$3y = 6$$

$$y = 2$$

$$3y = 6$$

$$y = 2$$

$$y = 2$$

$$y = 2$$

$$y = 2$$

$$y = 2$$

Y-intercept: $(0, 2)$

X-Intercept

The x-intercept is where the line crosses the x-axis (when $y = 0$).

To find: Set $y = 0$ and solve for x .

 **Example**

Find x-intercept of [Math: $2x + 3y = 6$]:

Set [Math: $y = 0$]:

[MathBlock]

$$2x + 3(0) = 6$$

[/MathBlock]

[MathBlock]

$$2x = 6$$

[/MathBlock]

[MathBlock]

$$x = 3$$

[/MathBlock]

X-intercept: [Math: $(3, 0)$]

 **Remember**

To find intercepts, substitute ZERO for the other variable:

- Y-intercept: Set [Math: $x = 0$], solve for [Math: y]
- X-intercept: Set [Math: $y = 0$], solve for [Math: x]

These are always single points (unless the line doesn't cross that axis, which is rare for linear functions).

Part 4: Properties of Linear Functions

Parallel Lines — Same Slope

Two lines are **parallel** if they have the same slope and different y-intercepts.

$$y = m x + c_1 \text{ and } y = m x + c_2 \text{ (where } c_1 \neq c_2)$$

Parallel lines never intersect.

Example

Lines $y = 2x + 3$ and $y = 2x - 5$ are parallel:

Both have slope $m = 2$, but different y-intercepts (3 and -5).

```
<JSXGraph id="rel-parallel" title="Parallel lines: y = 2x + 1 and y = 2x - 2" boundingbox=[[-4, 6, 4, -6]] height={320} code="board.create('functiongraph',[function(x){return 2x+1;},-4,4],{strokeColor:'#2563eb',strokeWidth:2,name:'y=2x+1',withLabel:true});board.create('functiongraph', [function(x){return 2x-2;},-4,4],{strokeColor:'#dc2626',strokeWidth:2,name:'y=2x-2',withLabel:true});" />
```

Example

Find a line parallel to $3x + 2y = 7$ passing through $(1, 4)$:

Step 1: Find the slope of $3x + 2y = 7$

$2y = -3x + 7$

$y = -\frac{3}{2}x + \frac{7}{2}$

Slope: $m = -\frac{3}{2}$

$y = -\frac{3}{2}x + \frac{7}{2}$

$y = -\frac{3}{2}x + \frac{7}{2}$

Slope: $m = -\frac{3}{2}$

Step 2: Parallel line has same slope: $m = -\frac{3}{2}$

Step 3: Use point-slope form with $(1, 4)$

$y - 4 = -\frac{3}{2}(x - 1)$

$y - 4 = -\frac{3}{2}x + \frac{3}{2}$

$y - 4 = -\frac{3}{2}x + \frac{3}{2}$

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$y = -\frac{3}{2}x + \frac{11}{2}$

$y = -\frac{3}{2}x + \frac{11}{2}$

$y = -\frac{3}{2}x + \frac{11}{2}$

Perpendicular Lines — Negative Reciprocal Slopes

Two lines are **perpendicular** if their slopes are negative reciprocals of each other.

If line 1 has slope m_1 and line 2 has slope m_2 :

$$m_1 \times m_2 = -1$$

Or:

$$m_2 = -\frac{1}{m_1}$$

Perpendicular lines meet at a 90° angle.

Examples:

- Slope 2 and slope $-\frac{1}{2}$ are perpendicular (because $2 \times (-\frac{1}{2}) = -1$)
- Slope 3 and slope $-\frac{1}{3}$ are perpendicular
- Slope $\frac{3}{4}$ and slope $-\frac{4}{3}$ are perpendicular

Example

Find a line perpendicular to $y = 2x + 5$ passing through $(3, -1)$:

Step 1: Slope of given line: $m_1 = 2$

Step 2: Perpendicular slope: $m_2 = -\frac{1}{2}$ (negative reciprocal)

Step 3: Use point-slope form with $(3, -1)$

$$[MathBlock]$$

$$y - (-1) = -\frac{1}{2}(x - 3)$$

$$[/MathBlock]$$

$$[MathBlock]$$

$$y + 1 = -\frac{1}{2}x + \frac{3}{2}$$

$$[/MathBlock]$$

$$[MathBlock]$$

$$y = -\frac{1}{2}x + \frac{1}{2}$$

$$[/MathBlock]$$

Length and Midpoint of Line Segments

Distance (Length) Formula

For two points (x_1, y_1) and (x_2, y_2) , the distance between them is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

This comes from the Pythagorean theorem.

Example

Find the distance between $(1, 2)$ and $(4, 6)$:

$$d = \sqrt{(4-1)^2 + (6-2)^2}$$

$$= \sqrt{3^2 + 4^2}$$

$$= \sqrt{9 + 16}$$

$$= \sqrt{25}$$

$$= 5 \text{ units}$$

Midpoint Formula

The midpoint of a line segment between (x_1, y_1) and (x_2, y_2) is:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Just average the x-coordinates and average the y-coordinates.

Example

Find the midpoint between $(2, 3)$ and $(8, 7)$:

$$\text{Midpoint} = \left(\frac{2+8}{2}, \frac{3+7}{2} \right) = \left(\frac{10}{2}, \frac{10}{2} \right) = (5, 5)$$

Check: Distance from $(2,3)$ to $(5,5)$ is $\sqrt{9+4} = \sqrt{13}$

Distance from $(5,5)$ to $(8,7)$ is $\sqrt{9+4} = \sqrt{13}$

Both equal, so $(5,5)$ is truly the midpoint.

Part 5: Graphing Linear Functions

Sketching from Slope-Intercept Form

Given [Math: $y = mx + c$]:

Step 1: Plot the y-intercept [Math: $(0, c)$]

Step 2: Use slope [Math: $m = \frac{\text{rise}}{\text{run}}$] to find more points:

- If [Math: $m = 2 = \frac{2}{1}$]: Go right 1, up 2
- If [Math: $m = -3 = \frac{-3}{1}$]: Go right 1, down 3
- If [Math: $m = \frac{2}{3}$]: Go right 3, up 2

Step 3: Plot at least 3 points and draw the line through them

Example

Sketch [Math: $y = -\frac{1}{2}x + 3$]:

- **Y-intercept:** [Math: $(0, 3)$] — start here
- **Slope:** [Math: $-\frac{1}{2} = \frac{-1}{2}$] (right 2, down 1)
- From [Math: $(0, 3)$]: go right 2, down 1 '[Math: $(2, 2)$]
- From [Math: $(2, 2)$]: go right 2, down 1 '[Math: $(4, 1)$]
- **Alternative direction** (left 2, up 1):
- From [Math: $(0, 3)$]: go left 2, up 1 '[Math: $(-2, 4)$]

`<JSXGraph id="rel-neg-half" title="y = -1/2x + 3" boundingbox=[-2, 5, 8, -2] height={320}`

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`-0.5*x+3;},-2,8),{strokeColor:'#2563eb',strokeWidth:2});board.create('point',[0,3],{name:'(0,3)',size:3,fillColor:c.`

`/>`

Solving Systems Graphically

When you have two linear equations, the **solution** is the point where the lines intersect.

Example

Solve graphically:

[MathBlock]

$$y = 2x - 1$$

[/MathBlock]

[MathBlock]

$$y = -x + 5$$

[/MathBlock]

Step 1: Graph both lines

- Line 1: y-intercept [Math: (0, -1)], slope 2
- Line 2: y-intercept [Math: (0, 5)], slope [Math: -1]

Step 2: Find intersection point

- From the graph: they intersect at [Math: (2, 3)]

Step 3: Verify

- Line 1: [Math: $y = 2(2) - 1 = 3$]
- Line 2: [Math: $y = -(2) + 5 = 3$]

Solution: [Math: (2, 3)]

```
<JSXGraph id="rel-intersect" title="y = x + 1 and y = -x + 3 intersect at (1, 2)" boundingbox=[[-3, 5, 5, -3]] height={320} code="board.create('functiongraph',[function(x){return x+1;},-3,5],{strokeColor:'#2563eb',strokeWidth:2,name:'y=x+1',withLabel:true});board.create('functiongraph',[function(x){return -x+3;},-3,5],{strokeColor:'#dc2626',strokeWidth:2,name:'y=-x+3',withLabel:true});board.create('point',[1,2],{name:'(1,2)'});" />
```