

# Linear Inequalities & Programming

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Linear programming uses inequalities to describe limits, then finds the best possible outcome inside those limits. The shaded region is not decoration; it represents all values that satisfy every condition at the same time.

For CSEC, these problems usually involve resources such as time, money, materials, or production limits. Define the variables, write the inequalities from the wording, shade the feasible region, and test corner points when maximising or minimising. Each step explains the model.

## Inequalities in One Variable

An inequality shows a relationship where something is greater than, less than, greater than or equal to, or less than or equal to something else.

Symbols:

- [Math: >] : greater than
- [Math: <] : less than
- [Math: \geq] : greater than or equal to
- [Math: \leq] : less than or equal to

## Solving One-Variable Linear Inequalities

Solve like equations, BUT **flip the inequality sign when multiplying or dividing by a negative number**.

**Example****Solve [Math:  $3x - 2 > 7$ ]:****Step 1:** Add 2 to both sides

[MathBlock]

$$3x > 9$$

[/MathBlock]

**Step 2:** Divide by 3 (positive, so don't flip)

[MathBlock]

$$x > 3$$

[/MathBlock]

**Answer:** All numbers greater than 3. Set notation: [Math:  $\{x : x > 3\}$ ]

Number line representation:

`<JSXGraph id="rel-xgt3" title="x > 3" boundingbox=[[-1, 1.5, 8, -1.5]] axis={false} height={140}``code="board.create('axis', [[-1,0],[8,0]],{ticks:{insertTicks:false,ticksDistance:1,minorTicks:0,majorHeight:8}});b``/>`**Example****Solve [Math:  $-2x + 5 \leq 1$ ]:****Step 1:** Subtract 5 from both sides

[MathBlock]

$$-2x \leq -4$$

[/MathBlock]

**Step 2:** Divide by -2 (negative, so FLIP the sign!)

[MathBlock]

$$x \geq 2$$

[/MathBlock]

**Answer:** All numbers greater than or equal to 2. Set notation: [Math:  $\{x : x \geq 2\}$ ]

Number line representation:

`<JSXGraph id="rel-xge2" title="x ≥ 2" boundingbox=[[-2, 1.5, 8, -1.5]] axis={false} height={140}``code="board.create('axis', [[-2,0],[8,0]],{ticks:{insertTicks:false,ticksDistance:1,minorTicks:0,majorHeight:8}});b``/>`

## Inequalities in Two Variables

An inequality like [Math:  $y > 2x + 1$ ] represents a region (not just a line).

## Graphing Two-Variable Inequalities

**Step 1:** Graph the boundary line [Math:  $y = 2x + 1$ ]

- Use dashed line if inequality is [Math:  $<$ ] or [Math:  $>$ ] (not included)
- Use solid line if inequality is [Math:  $\leq$ ] or [Math:  $\geq$ ] (included)

**Step 2:** Shade the appropriate region

- For [Math:  $y > 2x + 1$ ]: shade ABOVE the line
- For [Math:  $y < 2x + 1$ ]: shade BELOW the line
- Test a point if unsure: pick [Math:  $(0, 0)$ ] and check if it satisfies the inequality

### Example

**Graph [Math:  $y \leq -x + 4$ ]:**

**Step 1:** Graph boundary line [Math:  $y = -x + 4$ ]

- Y-intercept: [Math:  $(0, 4)$ ]
- Slope: [Math:  $-1$ ]
- Use SOLID line (inequality includes [Math:  $=$ ])

**Step 2:** Shade region

- Test point [Math:  $(0, 0)$ ]: [Math:  $0 \leq -(0) + 4 = 4$ ] TRUE
- Since [Math:  $(0, 0)$ ] makes it true and it's below the line, shade BELOW

```
<JSXGraph id="rel-ylex" title="Region y dx + 4 (shaded below the line)" boundingbox={[-2, 6, 6, -2]} height={340}
```

```
code="board.create('inequality',[board.create('line',[[0,4],[4,0]],{strokeColor:'#2563eb',strokeWidth:2})),{invers
/>
```

## What Is Linear Programming?

**Linear programming** is a method to find the best solution (maximum or minimum) when you have:

- An **objective function** (what you want to maximize or minimize)
- **Constraints** (limitations or restrictions, usually linear inequalities)

**Real-world applications:**

- Maximize profit subject to resource limits
- Minimize cost subject to production requirements
- Optimize resource allocation

## The Feasible Region

The **feasible region** is the area that satisfies all constraints simultaneously.

### To find it:

- 1. Graph each constraint as a linear inequality
- 2. Shade the region that satisfies ALL constraints
- 3. The overlapping region is the feasible region

### Example

#### Graph the constraints:

$x + y \leq 8$

$x + y \leq 8$

$x + y \leq 8$

$2x + y \leq 12$

$2x + y \leq 12$

$2x + y \leq 12$

$x \geq 0, y \geq 0$

$x \geq 0, y \geq 0$

$x \geq 0, y \geq 0$

#### Step 1: Graph each line

- Line 1:  $x + y = 8$  (intercepts at  $(0, 8)$  and  $(8, 0)$ )
- Line 2:  $2x + y = 12$  (intercepts at  $(0, 12)$  and  $(6, 0)$ )
- Lines 3 & 4: axes (since  $x \geq 0$  and  $y \geq 0$ )

#### Step 2: Shade regions below/right of lines for $\leq$ and $\geq$

#### Step 3: Overlapping region = feasible region (usually a polygon)

```
<JSXGraph id="rel-lp" title="Feasible region (intersection of  $x+y=8$ ,  $x=0$ ,  $y=0$ )" boundingbox=[-1, 7, 8, -1] height=360 code="var
```

```
l1=board.create('line',[[0,6],[6,0]],{strokeColor:'#2563eb',strokeWidth:2,name:'x+y=6',withLabel:true});var
```

```
l2=board.create('line',[[0,4],[1,4]],{strokeColor:'#dc2626',strokeWidth:2,name:'y=4',withLabel:true});board.crea
```

```
/>
```

## Optimization — Finding the Best Solution

The **optimal solution** occurs at a **vertex (corner point)** of the feasible region.

### Method:

- 1. Find all corner points of the feasible region
- 2. Evaluate the objective function at each corner point

- 3. Choose the point that gives the maximum or minimum value

### Example

**Maximize profit:**  $P = 3x + 2y$  subject to:

$x + y \leq 8$

$x + y \leq 8$

$2x + y \leq 12$

$2x + y \leq 12$

$x \geq 0, y \geq 0$

$x \geq 0, y \geq 0$

$x \geq 0, y \geq 0$

**Step 1:** Find corner points of feasible region

- $(0, 0)$  (origin)
- $(6, 0)$  (where line 2 meets x-axis)
- $(4, 4)$  (where lines 1 and 2 intersect)
- $(0, 8)$  (where line 1 meets y-axis)

**Step 2:** Evaluate  $P = 3x + 2y$  at each corner

- At  $(0, 0)$ :  $P = 3(0) + 2(0) = 0$
- At  $(6, 0)$ :  $P = 3(6) + 2(0) = 18$
- At  $(4, 4)$ :  $P = 3(4) + 2(4) = 12 + 8 = 20$  •MAXIMUM
- At  $(0, 8)$ :  $P = 3(0) + 2(8) = 16$

**Optimal solution:**  $(4, 4)$  gives maximum profit of 20

This means: produce 4 of product  $x$  and 4 of product  $y$  for maximum profit.