Sectors, Surface Area & Volume

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Measurement questions turn shapes into quantities: lengths, areas, surface areas, and volumes. The main challenge is deciding which part of the object is being measured. An arc is a length, a sector is an area, surface area covers the outside, and volume fills the inside.

CSEC Paper 02 often gives measurement questions with diagrams and real-world wording. Read the units carefully, identify whether the answer should be linear, square, or cubic units, and avoid rounding until the final line. Clear unit use is part of the explanation, not an extra decoration.

Arc Length

Arc length is a fraction of the full circumference. The central angle tells you what fraction of the circle you are using.

An **arc** is part of a circle's circumference.

$$\text{Arc length} = \frac{\theta}{360^\circ} \times 2\pi r$$

Where θ = angle at center (in degrees).

> 📋 **Example**
> **Circle with radius 6 cm, arc with central angle 60°:**
>
> $$\text{Arc} = \frac{60^\circ}{360^\circ} \times 2\pi(6) = \frac{1}{6} \times 12\pi = 2\pi \approx 6.28 \text{ cm}$$

Area of a Sector

A sector is a slice of a circle, so its area is the same fraction of the whole circle's area as its central angle is of 360°.

A **sector** is a "pizza slice" of a circle.

$$\text{Sector area} = \frac{\theta}{360^\circ} \times \pi r^2$$

📋 **Example**

Circle with radius 8 cm, sector with central angle 90°:

$$A = \frac{90^\circ}{360^\circ} \times \pi(8)^2 = \frac{1}{4} \times 64\pi = 16\pi \approx 50.3 \text{ cm}^2$$

Area of a Segment

A segment is not the same as a sector. It is the curved cap left after removing the triangle formed by the two radii and the chord.

A **segment** is the area between a chord and the arc (NOT including the triangle).

$$\text{Segment area} = \text{Sector area} - \text{Triangle area}$$

📋 **Example**

Circle radius 5 cm, central angle 60°:

Sector area: $\frac{60^\circ}{360^\circ} \times \pi(5)^2 = \frac{1}{6} \times 25\pi \approx 13.09$ cm²

Triangle area (isosceles with two sides = 5 cm, angle = 60°):

$$A = \frac{1}{2} \times 5 \times 5 \times \sin(60^\circ) = 12.5 \times 0.866 \approx 10.83 \text{ cm}^2$$

Segment area: $13.09 - 10.83 = 2.26$ cm²

Surface Area of 3D Shapes

Surface area counts the outside faces only. Imagine unfolding the solid into a net and adding the areas of all exposed faces.

Surface area is the total area of ALL faces of a 3D shape.

Cube and Cuboid

Cube (all sides equal):

$$SA = 6s^2$$

Cuboid (rectangular box):

$$SA = 2(lw + lh + wh)$$

📋 **Example**
Cube with side 4 cm:

$$SA = 6(4)^2 = 6 \times 16 = 96 \text{ cm}^2$$

📋 **Example**
Cuboid: length 8 cm, width 5 cm, height 3 cm:

$$SA = 2(8 \times 5 + 8 \times 3 + 5 \times 3)$$

$$SA = 2(40 + 24 + 15) = 2(79) = 158 \text{ cm}^2$$

Prism

A prism has the same cross-section all the way through. Its surface area combines the two identical ends with the rectangular faces around the sides.

A **prism** has two identical parallel faces (bases) and rectangular sides.

$$SA = 2 \times (\text{base area}) + (\text{perimeter of base}) \times \text{height}$$

📋 **Example**

Triangular prism: triangular base with sides 3, 4, 5 cm; prism height 10 cm:

Base area (triangle): $\frac{1}{2} \times 3 \times 4 = 6\,\text{cm}^2$

Base perimeter: $3 + 4 + 5 = 12\,\text{cm}$

$$SA = 2(6) + 12 \times 10 = 12 + 120 = 132 \text{ cm}^2$$

Cylinder

A cylinder's surface area comes from two circular ends plus one curved rectangle wrapped around the side.

$$SA = 2\pi r^2 + 2\pi rh$$

Or: $SA = 2\pi r(r + h)$

Where the first term is the two circular bases, the second is the curved surface.

📋 **Example**

Cylinder: radius 3 cm, height 8 cm:

$$SA = 2\pi(3)^2 + 2\pi(3)(8)$$

$$SA = 2\pi(9) + 2\pi(24) = 18\pi + 48\pi = 66\pi \approx 207.3 \text{ cm}^2$$

Cone

A cone's surface area combines the circular base with the curved slant surface. Use slant height for surface area, not vertical height.

$$SA = \pi r^2 + \pi rl$$

Where l = slant height (NOT the perpendicular height).

📋 **Example**

Cone: radius 4 cm, slant height 10 cm:

$$SA = \pi(4)^2 + \pi(4)(10) = 16\pi + 40\pi = 56\pi \approx 175.8 \text{ cm}^2$$

Sphere

A sphere has no flat faces, so its surface area uses a special formula based only on radius.

$$SA = 4\pi r^2$$

📋 **Example**

Sphere: radius 5 cm:

$$SA = 4\pi(5)^2 = 4\pi(25) = 100\pi \approx 314.2 \text{ cm}^2$$

Volume of 3D Shapes

Volume measures the space inside a solid, so answers use cubic units. Many formulas are built from the idea: area of base multiplied by height, with a fraction added for pointed solids.

Volume is the amount of space inside a 3D shape. It's measured in cubic units (cm^3, m^3, etc.).

Cube and Cuboid

For cuboids, volume counts layers of rectangular area stacked through the height. For cubes, all three dimensions are equal.

Cube:

$$V = s^3$$

Cuboid:

$$V = l \times w \times h$$

📋 **Example**
Cube: side 5 cm:

$$V = 5^3 = 125\ \text{cm}^3$$

📋 **Example**
Cuboid: 10 cm × 6 cm × 4 cm:

$$V = 10 \times 6 \times 4 = 240\ \text{cm}^3$$

Prism

Any prism keeps the same cross-section throughout, so volume is cross-sectional area multiplied by length.

$$V = (\text{base area}) \times \text{height}$$

📋 **Example**
Triangular prism: triangular base area 12 cm², prism height 8 cm:

$$V = 12 \times 8 = 96\ \text{cm}^3$$

Cylinder

A cylinder is a circular prism. Its base area is πr^2
, and the height tells how many circular layers are stacked.

$$V = \pi r^2 h$$

📋 **Example**
Cylinder: radius 4 cm, height 10 cm:

$$V = \pi(4)^2(10) = \pi(16)(10) = 160\pi \approx 502.7\ \text{cm}^3$$

Pyramid

A pyramid has one third the volume of a prism with the same base area and height. This is why the formula includes
$\frac{1}{3}$.

$$V = \frac{1}{3} \times (\text{base area}) \times \text{height}$$

📋 **Example**

Pyramid: square base 6 cm × 6 cm, height 9 cm:

Base area = $6^2 = 36$ cm²

$$V = \frac{1}{3} \times 36 \times 9 = \frac{1}{3} \times 324 = 108 \text{ cm}^3$$

Cone

A cone has one third the volume of a cylinder with the same base radius and height.

$$V = \frac{1}{3}\pi r^2 h$$

📋 **Example**

Cone: radius 5 cm, height 12 cm:

$$V = \frac{1}{3}\pi(5)^2(12) = \frac{1}{3}\pi(25)(12) = \frac{1}{3}(300\pi) = 100\pi \approx 314.2 \text{ cm}^3$$

Sphere

Sphere volume depends on radius in three dimensions, so the radius is cubed. A small change in radius can make a large change in volume.

$$V = \frac{4}{3}\pi r^3$$

📋 **Example**

Sphere: radius 6 cm:

$$V = \frac{4}{3}\pi(6)^3 = \frac{4}{3}\pi(216) = \frac{864}{3}\pi = 288\pi \approx 904.8\ \text{cm}^3$$

🧠 **Remember**

Volume quick reference:

- Prism/Cylinder: Base area × height
- Pyramid/Cone: S× Base area × height
- Sphere: ƒÀ^3