

# Number Bases & Binary

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Number bases show that place value depends on the system being used. In base 10, each place is a power of 10; in base 2, each place is a power of 2; in base 5, each place is a power of 5.

CSEC questions on bases test careful conversion, not guesswork. Write the place values, expand the number, and then regroup if converting from base 10 to another base. Remember that a digit in a base must be smaller than the base itself.

So far, we've been working in **base 10** (the decimal system) because we have 10 fingers. But numbers can be represented in ANY base, and computers use **base 2** (binary).

## Understanding Place Value in Different Bases

In base 10, we use digits 0-9, and each place value represents a power of 10:

Thousands	Hundreds	Tens	Ones
$10^3$	$10^2$	$10^1$	10p
1000	100	10	1

So the number 2,345 in base 10 means:


$$2345_{10} = 2(1000) + 3(100) + 4(10) + 5(1)$$

In **base 2** (binary), we use only digits 0 and 1, and each place value represents a power of 2:

Sixteens	Eights	Fours	Twos	Ones
2t	$2^3$	$2^2$	$2^1$	2p
16	8	4	2	1

## Converting FROM Binary TO Decimal (Base 2 to Base 10)

This is the **table method**, write out place values in a table, then add up the values where there's a 1.

 **Example**

**Convert  $10110_2$  (binary) to decimal**

**Step 1:** Draw a table with powers of 2

$2^4$	$2^3$	$2^2$	$2^1$	$2^0$
16	8	4	2	1

**Step 2:** Write the binary digits in the table

$2^4$	$2^3$	$2^2$	$2^1$	$2^0$
1	0	1	1	0


**Step 3:** For each 1, take that place value. Ignore the 0s.

- Position  $2^4$  : digit is 1, so count 16
- Position  $2^3$  : digit is 0, so count 0
- Position  $2^2$  : digit is 1, so count 4
- Position  $2^1$  : digit is 1, so count 2
- Position  $2^0$  : digit is 0, so count 0

**Step 4:** Add them up

$$16 + 0 + 4 + 2 + 0 = 22$$

**Answer:**  $10110_2 = 22_{10}$

 **Example**

**Convert  $11001_2$  to decimal**

$2^4$	$2^3$	$2^2$	$2^1$	$2^0$
16	8	4	2	1

Add the values where digit is 1:

$$16 + 8 + 0 + 0 + 1 = 25$$

**Answer:**  $11001_2 = 25_{10}$

## Converting FROM Decimal TO Binary (Base 10 to Base 2)

Use the **repeated division method**: Keep dividing by 2 and track remainders.

### Example

**Convert 22 (decimal) to binary**

**Step 1:** Divide repeatedly by 2, writing down remainders

[CodeBlock:0]

**Step 2:** Read the remainders from BOTTOM to TOP

**Remainders from bottom to top: 1, 0, 1, 1, 0**

**Answer:**  $22_{10} = 10110_2$

**Check:** Use the table method above to verify,  $10110_2 = 16 + 4 + 2 = 22$

### Example

**Convert 35 to binary**

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Read remainders from bottom to top: **100011**

**Answer:**  $35_{10} = 100011_2$


**Check:**  $32 + 2 + 1 = 35$

## Other Number Bases (Base 8 and Base 16)

While binary (base 2) is most important for computer science, other bases follow the same place-value principle.

**Base 8 (Octal):** Uses digits 0-7, each place is a power of 8

$8^3$	$8^2$	$8^1$	$8^0$
512	64	8	1

 **Example****Convert  $253_8$  (octal) to decimal**

$8^2$	$8^1$	$8^0$
-----	-----	-----
2	5	3
64	8	1

$$2(64) + 5(8) + 3(1) = 128 + 40 + 3 = 171_{10}$$

**Answer:**  $253_8 = 171_{10}$  **Exam Tip**

**For CSEC:** Focus on base 2 (binary), base 8 (octal), and base 10 (decimal). The method is always the same: draw the place value table, put in your digits, and add up using powers of the base.

## Properties of Numbers and Operations

Numbers and mathematical operations have special properties that help us solve problems more efficiently.

### Closure Property

A set is **closed** under an operation if performing that operation on numbers in the set always gives you a number also in that set.

**Addition:** Whole numbers are closed under addition (adding two whole numbers always gives a whole number)

- $3 + 4 = 7$

**Subtraction:** Whole numbers are NOT closed under subtraction ( $2 - 5 = -3$ , which is not a whole number)

**Multiplication:** Whole numbers are closed under multiplication

- $4 \times 6 = 24$

**Division:** Whole numbers are NOT closed under division ( $5 \div 2 = 2.5$ , which is not a whole number)

## Commutative Property

The order doesn't matter, you get the same answer either way.

**Addition:**  $a + b = b + a$

- $3 + 5 = 8$  and  $5 + 3 = 8$

**Multiplication:**  $a \times b = b \times a$

- $4 \times 7 = 28$  and  $7 \times 4 = 28$

**Subtraction:** NOT commutative ( $5 - 3 \neq 3 - 5$ )

**Division:** NOT commutative ( $8 \div 4 \neq 4 \div 8$ )

## Associative Property

When you have multiple operations, you can group them differently and get the same answer.

**Addition:**  $(a + b) + c = a + (b + c)$

- $(2 + 3) + 4 = 5 + 4 = 9$
- $2 + (3 + 4) = 2 + 7 = 9$

**Multiplication:**  $(a \times b) \times c = a \times (b \times c)$

- $(2 \times 3) \times 4 = 6 \times 4 = 24$
- $2 \times (3 \times 4) = 2 \times 12 = 24$

**Subtraction:** NOT associative

**Division:** NOT associative

## Distributive Property

Multiplication distributes over addition:

$$a \times (b + c) = (a \times b) + (a \times c)$$

**Example**

Calculate  $5 \times (3 + 4)$  two ways:

**Method 1** (do the parentheses first):

$$5 \times (3 + 4) = 5 \times 7 = 35$$

**Method 2** (distribute the 5):

$$5 \times (3 + 4) = (5 \times 3) + (5 \times 4) = 15 + 20 = 35$$

Both give 35!

## Identity Properties

**Additive Identity:** Adding 0 doesn't change the number.

$$a + 0 = a$$

**Multiplicative Identity:** Multiplying by 1 doesn't change the number.

$$a \times 1 = a$$

## Inverse Properties

**Additive Inverse:** For any number, there's another number that, when added, gives 0.

$$a + (-a) = 0$$

- $5 + (-5) = 0$

**Multiplicative Inverse:** For any non-zero number, there's another number that, when multiplied, gives 1.

$$a \times \frac{1}{a} = 1$$

- $5 \times \frac{1}{5} = 1$