Sequences, Factors & Multiples

Amari Cross & Matthew Williams • Math • May 6, 2026

Number theory looks for structure inside whole numbers. Sequences reveal patterns, factors break numbers into parts, and multiples show repeated groups.

In CSEC, these ideas may appear as direct Paper 01 questions or as reasoning steps inside larger problems. For HCF, think "largest shared factor"; for LCM, think "first shared multiple". For sequences, describe the rule in words before writing a formula.

A **sequence** is a list of numbers that follow a pattern or rule. Finding the pattern is like solving a puzzle!

Types of Sequences

Arithmetic Sequences (also called linear sequences): Each term increases or decreases by the SAME amount.

Example: 2, 5, 8, 11, 14, ...

Pattern: Each term is 3 more than the previous term.

Position	**1st**	**2nd**	**3rd**	**4th**	**5th**
Term	2	5	8	11	14
Difference	—	+3	+3	+3	+3

The difference between consecutive terms is called the **common difference**, often written as **d**.

Formula for arithmetic sequences:

$$a_n = a_1 + (n-1)d$$

Where:

- [Math: a_n] = the nth term (the term you're looking for)
- [Math: a_1] = the first term
- [Math: n] = which position (1st, 2nd, 3rd, etc.)
- [Math: d] = the common difference

📋 **Example**

Find the 10th term of the sequence: 2, 5, 8, 11, 14, ...

Step 1: Identify what we know

- First term: [Math: a_1 = 2]
- Common difference: [Math: d = 3]
- We want: [Math: a_{10}] (the 10th term)
- We want to find: [Math: n = 10]

Step 2: Use the formula

[MathBlock]
an = a1 + (n-1)d
[/MathBlock]

[MathBlock]
a_{10} = 2 + (10-1)(3)
[/MathBlock]

[MathBlock]
a_{10} = 2 + (9)(3)
[/MathBlock]

[MathBlock]
a_{10} = 2 + 27
[/MathBlock]

[MathBlock]
a_{10} = 29
[/MathBlock]

Answer: The 10th term is 29.

Finding the Pattern

Sometimes you're given a sequence and need to figure out the rule.

📋 **Example**

What's the rule for this sequence? 5, 10, 15, 20, 25, ...

Method: Look at the differences

- 10 - 5 = 5
- 15 - 10 = 5
- 20 - 15 = 5
- 25 - 20 = 5

Pattern: It's an arithmetic sequence with common difference d = 5.

Rule: Multiply the position by 5.

- Position 1: 5 × 1 = 5
- Position 2: 5 × 2 = 10
- Position 3: 5 × 3 = 15
- Position n: 5n

Answer: The nth term = 5n

Factors and Multiples

These are two closely related ideas that show up constantly in math and especially in number theory.

Understanding Factors

A **factor** of a number is any number that divides into it evenly (with no remainder).

In other words: If [Math: a ÷ b] gives you a whole number, then [Math: b] is a factor of [Math: a].

Example: What are the factors of 12?

Let's check each number:

- 12 ÷ 1 = 12 (1 is a factor)
- 12 ÷ 2 = 6 (2 is a factor)
- 12 ÷ 3 = 4 (3 is a factor)
- 12 ÷ 4 = 3 (4 is a factor)
- 12 ÷ 5 = 2.4 (5 is NOT a factor)
- 12 ÷ 6 = 2 (6 is a factor)
- 12 ÷ 12 = 1 (12 is a factor)

Factors of 12: \{1, 2, 3, 4, 6, 12\}

🧠 **Remember**

Key facts about factors:

- Every number has at least 2 factors: 1 and itself
- 1 is a factor of every number
- The number itself is always a factor of itself
- Factors are always dthe original number

Understanding Multiples

A **multiple** of a number is the result of multiplying that number by any other whole number.

In other words: If you can write [Math: a = b × c] (where [Math: c] is a whole number), then [Math: a] is a multiple of [Math: b].

Example: What are some multiples of 5?

Multiply 5 by different whole numbers:

- $5 \times 1 = 5$
- $5 \times 2 = 10$
- $5 \times 3 = 15$
- $5 \times 4 = 20$
- $5 \times 5 = 25$
- $5 \times 6 = 30$

Some multiples of 5: \{5, 10, 15, 20, 25, 30, ...\}

🧠 **Remember**

Key facts about multiples:

- Every number is a multiple of itself
- 0 is a multiple of every number (since any number × 0 = 0)
- You can generate infinite multiples — keep multiplying!
- Multiples are always ethe original number
- The first multiple of a number is the number itself

Factor vs. Multiple (They're Opposites!)

These concepts are closely related — in fact, they're OPPOSITES:

If **2 is a factor of 12**, then **12 is a multiple of 2**.

If **3 is a factor of 15**, then **15 is a multiple of 3**.

Think of it this way:

- **Factors**: What divides INTO the number?
- **Multiples**: What does the number divide INTO?

> 📋 Example
> **For the number 4:**
>
> **Factors** (what divides into 4): \{1, 2, 4\}
> **Multiples** (what 4 divides into): \{4, 8, 12, 16, 20, 24, ...\}

Prime and Composite Numbers

Now we're going to classify numbers based on how many factors they have.

Prime Numbers

A **prime number** is a natural number greater than 1 that has EXACTLY two factors: 1 and itself.

Why greater than 1? Because 1 has only one factor (itself), so mathematicians decided not to call it prime.

The first few prime numbers: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, ...

Why are these prime?

- 2: Factors are \{1, 2\} — only 2 factors
- 3: Factors are \{1, 3\} — only 2 factors
- 5: Factors are \{1, 5\} — only 2 factors
- 7: Factors are \{1, 7\} — only 2 factors

> 🧠 Remember
> **Special fact**: 2 is the ONLY even prime number. All other prime numbers are odd. (All other even numbers can be divided by 2, so they have at least 3 factors: 1, 2, and itself.)

Composite Numbers

A **composite number** is a natural number greater than 1 that has MORE than two factors.

Example composite numbers: 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, ...

Why are these composite?

- 4: Factors are \{1, 2, 4\} — more than 2 factors
- 6: Factors are \{1, 2, 3, 6\} — more than 2 factors
- 9: Factors are \{1, 3, 9\} — more than 2 factors
- 10: Factors are \{1, 2, 5, 10\} — more than 2 factors

📋 **Example**

Is 17 prime or composite?

Check the factors:

- 17 ÷ 1 = 17
- 17 ÷ 2 = 8.5
- 17 ÷ 3 = 5.67...
- 17 ÷ 4 = 4.25
- We only need to check up to about [Math: \sqrt{17}] ≈4.1

Factors of 17: \{1, 17\}

Answer: 17 is prime (only 2 factors)

📝 **Exam Tip**

Quick tip for testing if a number is prime: Only test dividing by prime numbers up to the square root of your number. For example, to test if 37 is prime, only check 2, 3, 5 (since [Math: \sqrt{37}] ≈6).

Highest Common Factor (HCF) and Lowest Common Multiple (LCM)

These two concepts show up in solving problems about fractions, ratios, and many other areas.

Highest Common Factor (HCF)

The **HCF** (also called GCD — Greatest Common Divisor) is the **largest number that divides evenly into two (or more) numbers**.

Think of it as the biggest factor they have in common.

Method 1: Listing Factors

📋 Example

Find the HCF of 12 and 18

Step 1: List all factors of each number

- Factors of 12: \{1, 2, 3, 4, 6, 12\}
- Factors of 18: \{1, 2, 3, 6, 9, 18\}

Step 2: Find the COMMON factors (factors they share)

- Common factors: \{1, 2, 3, 6\}

Step 3: Choose the HIGHEST one

- **HCF = 6**

Check: 12 ÷ 6 = 2 and 18 ÷ 6 = 3

Method 2: Prime Factorization (More powerful method)

Every number can be broken down into a product of prime numbers. This is called **prime factorization**.

📋 Example

Find the HCF of 24 and 36 using prime factorization

Step 1: Break each number into prime factors

- 24 = 2 × 12 = 2 × 2 × 6 = 2 × 2 × 2 × 3 = [Math: 2^3 × 3]
- 36 = 2 × 18 = 2 × 2 × 9 = 2 × 2 × 3 × 3 = [Math: 2^2 × 3^2]

Step 2: Identify COMMON prime factors with the SMALLEST powers

- Common prime: 2 (appears as [Math: 2^3] in 24 and [Math: 2^2] in 36 — use the smaller: [Math: 2^2])
- Common prime: 3 (appears as [Math: 3^1] in 24 and [Math: 3^2] in 36 — use the smaller: [Math: 3^1])

Step 3: Multiply these together

[MathBlock]
\text{HCF} = 2^2 × 3 = 4 × 3 = 12
[/MathBlock]

Check: 24 ÷ 12 = 2 and 36 ÷ 12 = 3

🧠 Remember

HCF rule: Take the LOWEST power of each COMMON prime factor.

Lowest Common Multiple (LCM)

The **LCM** is the **smallest number that is a multiple of two (or more) numbers**.

Think of it as the smallest multiple they have in common.

Method 1: Listing Multiples

> 📋 **Example**
> **Find the LCM of 4 and 6**
>
> **Step 1**: List multiples of each number
> - Multiples of 4: \{4, 8, 12, 16, 20, 24, ...\}
> - Multiples of 6: \{6, 12, 18, 24, 30, ...\}
>
> **Step 2**: Find COMMON multiples
> - Common multiples: \{12, 24, 36, ...\}
>
> **Step 3**: Choose the LOWEST one
> - **LCM = 12**
>
> **Check**: 12 ÷ 4 = 3 and 12 ÷ 6 = 2

Method 2: Prime Factorization

> 📋 **Example**
> **Find the LCM of 12 and 18 using prime factorization**
>
> **Step 1**: Break each number into prime factors
> - 12 = [Math: 2^2 × 3]
> - 18 = [Math: 2 × 3^2]
>
> **Step 2**: Identify ALL prime factors with the HIGHEST powers
> - Prime 2: appears as [Math: 2^2] in 12 and [Math: 2^1] in 18 — use the higher: [Math: 2^2]
> - Prime 3: appears as [Math: 3^1] in 12 and [Math: 3^2] in 18 — use the higher: [Math: 3^2]
>
> **Step 3**: Multiply these together
>
> [MathBlock]
> \text{LCM} = 2^2 × 3^2 = 4 × 9 = 36
> [/MathBlock]
>
> **Check**: 36 ÷ 12 = 3 and 36 ÷ 18 = 2

> 🧠 **Remember**
> **LCM rule**: Take the HIGHEST power of each prime factor that appears.

📝 **Exam Tip**

Real-world use: LCM is super useful! For example, if buses arrive every 4 minutes and trains every 6 minutes, they both arrive together every LCM(4,6) = 12 minutes.