Number Sets & Ordering

Matthew Williams • Math • May 6, 2026

Number sets help you classify values before you operate on them. Knowing whether a number is natural, whole, integer, rational, irrational, or real tells you what kind of answer is possible.

CSEC may ask you to identify numbers in a set, place them on a number line, or compare their sizes. When ordering numbers, convert them to a common form if needed, such as decimals or fractions. The goal is not just to name the set, but to understand how the numbers relate.

Before we can do anything with numbers, we need to understand the different **types** or **sets** of numbers. These are like different groups or families of numbers, and each one has specific properties.

Natural Numbers

Natural Numbers are the numbers you learned to count with as a child: **1, 2, 3, 4, 5, 6, ...**

Think of natural numbers as "counting numbers." If someone asked "how many apples are in a basket?" you couldn't answer "0" apples, you'd either say "1 apple" or "5 apples" or some other positive whole number. That's why natural numbers start at 1.

Symbol: We write this set as or \{1, 2, 3, 4, 5, ...\}

Real-world examples:

- The number of students in your class
- Goals scored in a football match
- Days in a month

Whole Numbers

Whole Numbers are natural numbers PLUS zero: **0, 1, 2, 3, 4, 5, ...**

The only difference between natural numbers and whole numbers is that **whole numbers include 0**. This might seem small, but zero is actually revolutionary! It allows us to represent "nothing" as a number.

Symbol: We write this set as **W** or \{0, 1, 2, 3, 4, ...\}

Why does 0 matter?

- If you have 0 dollars in your bank account, you need a number to represent that

- In temperature, 0 is meaningful (like 0°C being freezing)
- In place value, 0 is essential (like the 0 in 105)

> 💡 Tip
> **Relationship**: Every natural number is a whole number, but not every whole number is a natural number (because 0 is a whole number but not a natural number). We write this as: **,W** (natural numbers are a subset of whole numbers)

Integers

Integers are whole numbers PLUS negative numbers: **..., -3, -2, -1, 0, 1, 2, 3, ...**

Integers include positive numbers, negative numbers, and zero. Now we can represent things that go in both directions from zero.

Symbol: We write this set as $or \{..., -2, -1, 0, 1, 2, ...\}

Real-world examples:

- Temperature: -5°C is 5 degrees below zero
- Bank account: -100 means you owe 100
- Altitude: -100 meters below sea level
- Time: -2 hours means 2 hours before some starting point

> 🧠 Remember
> **Relationship**: Every whole number is an integer, but not every integer is a whole number. **W ,$**
> So far we have: **,W ,$**

Rational Numbers

Rational Numbers are any numbers that can be written as a fraction of two integers.

A rational number is any number of the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$ (we can't divide by zero).

Symbol: We write this set as

What counts as a rational number?

- All integers: 5 = $\frac{5}{1}$, -3 = $\frac{-3}{1}$, 0 = $\frac{0}{1}$
- All fractions: $\frac{3}{4}$, $\frac{7}{2}$, $\frac{-5}{8}$

$\frac{1}{2}$ $\frac{3}{4}$ $\frac{5}{2}$

- All decimals that terminate: 0.5 = , 0.75 = , 2.5 =
- All decimals that repeat: 0.333... = $\frac{1}{3}$, 0.666... = $\frac{2}{3}$

📋 **Example**

Which of these are rational numbers?

- 7 (because 7 = $\frac{7}{1}$)
- -4.5 (because -4.5 = $\frac{-9}{2}$)
- $\frac{2}{3}$ (already a fraction)
- 0.123123123... (repeating) (this equals $\frac{41}{333}$)
- π (this goes on forever WITHOUT repeating, see irrational numbers)

🧠 **Remember**

Relationship: Every integer is a rational number. **$**

So far: **,W ,$**

Irrational Numbers

Irrational Numbersare real numbers that CANNOT be written as a fraction $\frac{p}{q}$.

These are numbers whose decimal representations go on forever WITHOUT repeating in a pattern.

Common irrational numbers:

- π H3.14159265... (the ratio of a circle's circumference to diameter)
- $\sqrt{2}$ H1.41421356... (the square root of 2)
- $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$ (square roots of non-perfect squares)

Why are these "irrational"?

The word "irrational" doesn't mean "crazy", it means "not a ratio." These numbers cannot be expressed as the ratio (fraction) of two integers.

📋 **Example**

$\sqrt{2}$ is irrational

If we try to write $\sqrt{2}$ as a decimal: $\sqrt{2}$ = 1.41421356237...

This goes on forever without repeating. We can never write it as a simple fraction. Try it on your calculator, keep pressing for more decimal places, and the digits never settle into a repeating pattern.

Real Numbers

Real Numbers are ALL the numbers above combined: rational numbers PLUS irrational numbers.

If we can plot it on a number line, it's a real number.

Symbol: We write this set as

🧠 **Remember**

The Complete Relationship:

$$\mathbb{N} \subset \mathbb{W} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$$

- Natural numbers: \{1, 2, 3, ...\}
- Whole numbers: \{0, 1, 2, 3, ...\}
- Integers: \{..., -2, -1, 0, 1, 2, ...\}
- Rational numbers: All fractions and their decimal equivalents
- Real numbers: Everything you can plot on a number line

Special Types of Numbers

Within these sets, several special categories of numbers are worth noting:

Square Numbers (Perfect Squares): Numbers that are the result of multiplying an integer by itself.

- 1 = 1 × 1
- 4 = 2 × 2
- 9 = 3 × 3
- 16 = 4 × 4
- 25 = 5 × 5

Even Numbers: Integers divisible by 2: \{..., -4, -2, 0, 2, 4, 6, ...\}

Odd Numbers: Integers NOT divisible by 2: \{..., -3, -1, 1, 3, 5, ...\}

Prime Numbers: Natural numbers greater than 1 that have ONLY two factors: 1 and itself. (More on this later!)

Composite Numbers: Natural numbers greater than 1 that have MORE than two factors.

Ordering and Comparing Real Numbers

A set of numbers can be arranged in order from smallest to largest (ascending) or from largest to smallest (descending).

Using the Number Line

The number line is your best friend here. Numbers further to the LEFT are smaller. Numbers further to the RIGHT are larger.

[Code: plaintext]

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📊 Interactive Number Line - Drag the blue point around!
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You can drag the blue point along the number line to see how position relates to value. The red axis shows all the integers, and notice how each tick mark is 1 unit apart.

Symbols for Comparing

We use special symbols to compare numbers:

- < means "less than" (smaller)
- > means "greater than" (larger)
- **=** means "equal to"
- **d**means "less than or equal to"
- **e**means "greater than or equal to"

📋 **Example**

Order these numbers from smallest to largest:

3, -2, 0.5, -4, 2

Step 1: Look at the interactive number line below, the red points show where each number is:

[CodeBlock:1]

Numbers to the LEFT are smaller. Numbers to the RIGHT are larger.

Step 2: Write them in order from left to right:

$$-4 < -2 < 0.5 < 2 < 3$$

📝 **Exam Tip**

When ordering negative numbers, remember: **-10 is smaller than -2** because -10 is further LEFT on the number line. The bigger the negative number looks, the smaller it actually is!