Sets & Set Notation

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Sets give mathematics a precise way to talk about collections. The symbols may look small, but they prevent confusion about what belongs, what does not belong, and how groups are related.

CSEC Sets questions often test notation directly, then use that notation in Venn diagrams and word problems. Learn the language first: element, subset, universal set, complement, empty set, and cardinality. Once the notation is clear, the calculations become much easier to follow.

What is a Set?

A **set** is a well-defined collection of distinct objects called **elements** or **members**.

Key characteristics:

- Elements are **distinct** (no repeats)
- A set is **well-defined**: membership can always be determined unambiguously
- Order doesn't matter

📋 **Example**

Good sets (well-defined):

- The set of vowels: \{a, e, i, o, u\}
- The set of even numbers: \{2, 4, 6, 8, ...\}
- The set of months with 31 days

Not sets (not well-defined):

- "The set of tall people", what counts as tall?
- "The set of good books", good is subjective

Membership: Is Something IN the Set?

If an object is in a set, we say it's a **member** or **element** of that set.

Notation:

- $a \in A$ means "a is an element of set A" or "a belongs to A"
- $a \notin A$ means "a is NOT an element of set A"

> 📋 **Example**
> Let $A = \{1, 2, 3, 5, 7\}$
>
> - $3 \in A$ (3 is in the set)
> - $4 \notin A$ (4 is not in the set)
> - $7 \in A$ (7 is in the set)

Cardinality: How Many Elements?

The **cardinality** of a set is the number of elements in it.

Notation: $n(A)$ means "the cardinality of set A" or "the number of elements in A"

> 📋 **Example**
> Let $B = \{a, e, i, o, u\}$
>
> The cardinality is $n(B) = 5$ (there are 5 vowels)
>
> Let $C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
>
> The cardinality is $n(C) = 10$ (there are 10 numbers)

Finite vs. Infinite Sets

- **Finite set**: Has a limited number of elements
- **Infinite set**: Has unlimited elements

> 📋 **Example**
> **Finite sets**:
> - The set of students in your class
> - The set of days in a week: \{Mon, Tue, Wed, Thu, Fri, Sat, Sun\}
> - The set of planets in our solar system
>
> **Infinite sets**:
> - The set of natural numbers: \{1, 2, 3, 4, ...\}
> - The set of integers: \{..., -2, -1, 0, 1, 2, ...\}
> - The set of all real numbers

Special Sets

Empty Set (or Null Set):

- Contains NO elements
- Notation: $\emptyset$ or \{ \}
- Example: The set of even prime numbers greater than 2

Universal Set:

- Contains ALL elements being considered in a problem
- Notation: U
- Changes depending on context

📋 **Example**

If we're discussing "numbers in a classroom game," the universal set might be \{1, 2, 3, 4, 5, 6\}.

If we're discussing "letters of the alphabet," the universal set is \{a, b, c, ..., z\}.

In a specific problem, we're told what U is.

🧠 **Remember**

- Order doesn't matter in sets: \{1, 2, 3\} = \{3, 1, 2\}
- Duplicates aren't allowed: \{1, 2, 2, 3\} = \{1, 2, 3\}
- The empty set is a valid set
- Every set is a subset of the universal set U

Representing Sets

Sets can be written in three different ways. Choosing the right form makes problems easier!

Method 1: Listing/Roster Form

Write all elements inside curly brackets, separated by commas.

Use this when: The set is small and finite.

📋 **Example**

- Set of vowels: $A = \{a, e, i, o, u\}$
- Set of digits: $B = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
- Set of even numbers less than 10: $C = \{2, 4, 6, 8\}$

If a set is infinite but follows a pattern, use "..." to show it continues:

📋 **Example**

- Set of natural numbers: $\mathbb{N} = \{1, 2, 3, 4, 5, \dots\}$
- Set of all even numbers: $E = \{2, 4, 6, 8, 10, \dots\}$
- Set of multiples of 5: $M = \{5, 10, 15, 20, \dots\}$

Method 2: Set-Builder Notation

Describe the condition that elements must satisfy.

Format: $\{x : \text{condition}\}$ which reads "the set of all x such that [condition]"

Use this when: It's hard or impossible to list all elements.

📋 **Example**

- $\{x : x \text{ is a vowel}\}$ = the set of all x such that x is a vowel
- $\{x : x \in \mathbb{N} \text{ and } x < 10\}$ = natural numbers less than 10 = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}
- $\{x : x \text{ is even and } 5 < x < 15\}$ = \{6, 8, 10, 12, 14\}
- $\{x : x \in \mathbb{Z}\}$ = all integers

💡 **Tip**

Common symbols in set-builder notation:

- $\in$ means "is an element of"
- $\notin$ means "is not an element of"
- $<, >, \leq, \geq$ are inequality signs
- $\mathbb{N}$ = natural numbers, $\mathbb{Z}$ = integers, $\mathbb{Q}$ = rationals, $\mathbb{R}$ = reals
- 'means "and", (means "or"

Method 3: Descriptive Form

Describe the set in words.

Use this when: Communication matters more than formality.

📋 **Example**

- "The set of all even numbers between 1 and 20"
- "The set of months with exactly 31 days"
- "The set of factors of 12"

📝 **Exam Tip**

On CSEC exams, you might need to:

- Convert listing form to set-builder notation
- Convert set-builder notation to listing form
- Identify which form is most appropriate
- Count elements in a set using any representation

Relationships Between Sets

Sets can relate to each other in important ways. Understanding these relationships is crucial.

Subset: Is One Set Inside Another?

Set A is a **subset** of set B if EVERY element of A is also in B.

Notation: $A \subseteq B$ (read as "A is a subset of B")

📋 **Example**

Let $A = \{1, 3, 5\}$ and $B = \{1, 2, 3, 4, 5\}$

Is $A \subseteq B$?

- Is 1 in B? Yes
- Is 3 in B? Yes
- Is 5 in B? Yes

All elements of A are in B, so $A \subseteq B$

📋 **Example**

Let $C = \{2, 4, 6\}$ and $D = \{1, 2, 3, 4, 5\}$

Is $C \subseteq D$?

- Is 2 in D? Yes
- Is 4 in D? Yes
- Is 6 in D? No

Not all elements of C are in D, so $C \nsubseteq D$ (C is NOT a subset of D)

🧠 **Remember**

Important subset facts:

- Every set is a subset of itself: $A \subseteq A$
- The empty set is a subset of every set: $\emptyset \subseteq A$ (for any set A)
- If $A \subseteq B$ and $B \subseteq A$, then $A = B$ (the sets are equal)

Equal Sets vs. Equivalent Sets

Equal sets have exactly the same elements.

- Notation: $A = B$
- Example: \{1, 2, 3\} = \{3, 1, 2\}

Equivalent sets have the SAME NUMBER of elements (but not necessarily the same elements).

- Notation: $A \equiv B$ or "A and B are equivalent"
- Example: \{1, 2, 3\} and \{a, b, c\} are equivalent (both have 3 elements)

📋 **Example**

Let $A = \{1, 2, 3\}$, $B = \{3, 2, 1\}$, and $C = \{a, b, c\}$

- Are A and B equal? Yes: $A = B$ (same elements)
- Are A and C equal? No: $A \neq C$ (different elements)
- Are A and C equivalent? Yes: both have cardinality 3

Complement of a Set

The **complement**of set A is the set of ALL elements in the universal set U that are NOT in A.

Notation: A' or A^c (read as "A complement" or "not A")

$$A' = \{x : x \in U \text{ and } x \notin A\}$$

📋 **Example**

Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $A = \{2, 4, 6, 8, 10\}$ (even numbers)

Then $A' = \{1, 3, 5, 7, 9\}$ (odd numbers)

This is everything in U that's NOT in A.

🧠 **Remember**

- $A \cup A' = U$ (A and its complement together make the universal set)
- $A \cap A' = \emptyset$ (A and its complement have no overlap)
- $(A')' = A$ (the complement of a complement is the original set)

Subsets and Counting

Finding All Subsets

Every set has multiple subsets. For a set with n elements, there are exactly 2^n subsets (including the empty set and the set itself).

📋 **Example**

Let $A = \{1, 2\}$

All subsets of A:

- 1. $\emptyset$ (empty set)
- 2. $\{1\}$
- 3. $\{2\}$
- 4. $\{1, 2\}$ (the set itself)

Total: $2^2 = 4$ subsets

Notice: Every element can either be "in" or "out" of a subset. That's 2 choices per element, so $2 \times 2 = 4$ total.

📋 **Example**

Let $B = \{a, b, c\}$

Number of subsets: $2^3 = 8$

All subsets:

- 1. $\emptyset$
- 2. $\{a\}$
- 3. $\{b\}$
- 4. $\{c\}$
- 5. $\{a, b\}$
- 6. $\{a, c\}$
- 7. $\{b, c\}$
- 8. $\{a, b, c\}$

That's 8 subsets!

📝 **Exam Tip**

Finding subsets systematically:

- 0 elements: just $\emptyset$ (1 subset)
- 1 element: one subset for each element (3 subsets if 3 elements)
- 2 elements: all pairs (3 subsets if 3 elements)
- 3 elements: all triples (1 subset if 3 elements)

For n elements, always: $1 + n + \binom{n}{2} + \binom{n}{3} + \ldots = 2^n$