Set Operations & Venn Diagrams

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Set operations explain how groups overlap, combine, or exclude each other. A Venn diagram turns those relationships into regions you can count.

CSEC Paper 02 includes a compulsory Sets component, so this topic is worth practising carefully. Start from the most specific region, usually the intersection, then work outward. When a question says "or", "and", "only", or "not", translate those words into union, intersection, exclusive regions, or complement before calculating.

Sets can be combined using three main operations: union, intersection, and complement.

Intersection: What's IN BOTH?

Intersection is the mathematical version of "and". An element must satisfy both conditions to be placed in the intersection.

The **intersection** of sets A and B is the set of elements that are in BOTH A and B.

Notation: [Math: A \cap B] (read as "A intersect B" or "A and B")

A \cap B = \{x : x \in A \text{ and } x \in B\}

📋 **Example**

Let [Math: A = \{1, 2, 3, 4, 5\}] and [Math: B = \{3, 4, 5, 6, 7\}]

What's in BOTH sets?

- Is 1 in both? 1 is in A but not B
- Is 3 in both? 3 is in both A and B
- Is 4 in both? 4 is in both A and B
- Is 5 in both? 5 is in both A and B

Therefore: [Math: A \cap B = \{3, 4, 5\}]

Disjoint Sets: If [Math: A \cap B = \emptyset], the sets have NO elements in common and are called **disjoint**.

📋 **Example**

Let $A = \{1, 2, 3\}$ and $B = \{4, 5, 6\}$

$A \cap B = \emptyset$ (nothing is in both)

A and B are disjoint.

Union: What's IN EITHER?

Union is the mathematical version of inclusive "or". Include elements from the first set, the second set, and the overlap, but do not repeat elements.

The **union** of sets A and B is the set of elements that are in A OR B (or both).

Notation: $A \cup B$ (read as "A union B" or "A or B")

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

📋 **Example**

Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{3, 4, 5, 6, 7\}$

What's in A OR B (or both)?

- Everything in A: $\{1, 2, 3, 4, 5\}$
- Everything in B: $\{3, 4, 5, 6, 7\}$
- Combined (without repeats): $\{1, 2, 3, 4, 5, 6, 7\}$

Therefore: $A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$

The Formula: Counting Elements in Unions

Use the union formula when the question gives totals instead of listing elements. The subtraction fixes the double-counting that happens in the overlap.

When you combine two sets with union, you might double-count overlapping elements. This formula corrects for that:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Why subtract the intersection? Because we counted those overlapping elements twice.

📋 **Example**

Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{3, 4, 5, 6, 7\}$

- $n(A) = 5$ (5 elements in A)
- $n(B) = 5$ (5 elements in B)
- $A \cap B = \{3, 4, 5\}$, so $n(A \cap B) = 3$

Using the formula:

$$n(A \cup B) = 5 + 5 - 3 = 7$$

Verify by listing: $A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$ has 7 elements

🧠 **Remember**

Set operation properties:

- $A \cup A = A$ (union with itself is itself)
- $A \cap A = A$ (intersection with itself is itself)
- $A \cup B = B \cup A$ (union is commutative)
- $A \cap B = B \cap A$ (intersection is commutative)
- $A \cup \emptyset = A$ and $A \cap \emptyset = \emptyset$
- $A \cup U = U$ and $A \cap U = A$

Part 6: Venn Diagrams

Venn diagrams are visual representations of sets and their relationships. They're extremely useful for solving set problems!

Understanding Venn Diagram Basics

The rectangle represents the universal set, meaning everything being considered in the problem. Anything outside a circle but inside the rectangle still belongs to the overall situation.

A Venn diagram uses circles to represent sets. Elements inside the circle are in the set; elements outside are not.

<JSXGraph id="set-venn-1" title="Set A inside universal set U" boundingbox={[-5, 4, 5, -4]} axis={false} height={300} code="board.create('polygon',[[-4.5,-3.5],[4.5,-3.5],[4.5,3.5],[-4.5,3.5]],{borders:{strokeColor:fg},fillColor:c.fill
/>

Two Sets: Intersection and Union

In a two-set Venn diagram, every item belongs in exactly one region: only A, only B, both, or neither. This prevents double-counting.

When two circles overlap, the overlapping region represents the intersection.

<JSXGraph id="set-venn-2" title="Two overlapping sets A and B" boundingbox={[-6, 4, 6, -4]} axis={false} height={300} code="board.create('polygon',[[-5.5,-3.5],[5.5,-3.5],[5.5,3.5],[-5.5,3.5]],{borders:{strokeColor:fg},fillColor:c.fill
/>

> 📋 **Example**
> Let [Math: U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}], [Math: A = \{1, 2, 4, 7\}], and [Math: B = \{2, 4, 5, 8\}]
>
> **In the Venn diagram**:
> - Only in A: \{1, 7\}
> - In both A and B: \{2, 4\}
> - Only in B: \{5, 8\}
> - Outside both (neither A nor B): \{3, 6, 9, 10\}
>
> **From this diagram, we can see**:
> - [Math: A \cap B = \{2, 4\}]
> - [Math: A \cup B = \{1, 2, 4, 5, 7, 8\}]
> - [Math: A' = \{3, 5, 6, 8, 9, 10\}]
> - [Math: (A \cup B)' = \{3, 6, 9, 10\}]

Three Sets: More Complex Relationships

For three sets, start with the centre region first because it affects all pairwise overlaps. Then subtract outward until each region contains only what belongs there.

With three sets, there are more regions. Use a Venn diagram with three overlapping circles.

<JSXGraph id="set-venn-3" title="Three overlapping sets A, B, C" boundingbox={[-6, 6, 6, -5]} axis={false} height={340} code="board.create('polygon',[[-5.5,-4.5],[5.5,-4.5],[5.5,5.5],[-5.5,5.5]],{borders:{strokeColor:fg},fillColor:c.fill
/>

📋 **Example**

Let [Math: U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}]:

- [Math: A = \{1, 2, 3, 4, 5\}]
- [Math: B = \{3, 4, 5, 6, 7\}]
- [Math: C = \{4, 5, 6, 8, 9\}]

Key intersections:

- [Math: A \cap B \cap C = \{4, 5\}] (in all three)
- [Math: A \cap B = \{3, 4, 5\}] (in A and B)
- [Math: A \cap C = \{4, 5\}] (in A and C)
- [Math: B \cap C = \{4, 5, 6\}] (in B and C)

The regions:

- Only in A: \{1, 2, 3\}
- Only in B: \{7\}
- Only in C: \{8, 9\}
- In A and B but not C: \{3\}
- In A and C but not B: \{\} (empty)
- In B and C but not A: \{6\}
- In all three: \{4, 5\}
- Outside all: \{10\}

How to Draw a Venn Diagram

Step 1: Draw the universal set as a rectangle.

Step 2: Draw circles for each set inside the rectangle.

Step 3: Label the circles with set names.

Step 4: Place each element in the correct region.

Step 5: Identify all regions (not just the intersections!).

📝 **Exam Tip**

When working with Venn diagrams on exams:

- 1. Count carefully — each element goes in exactly ONE region
- 2. Don't forget elements outside all sets (but inside U)
- 3. Use the regions to find unions, intersections, and complements
- 4. Label regions clearly
- 5. Double-check your counts using [Math: n(A \cup B) = n(A) + n(B) - n(A \cap B)]

Part 7: Solving Problems with Venn Diagrams

This is where Venn diagrams shine. They help organize complicated information.

Problem Type 1: From Data to Venn Diagram

This problem type tests organisation. Put the "both" value into the overlap first, then subtract it from each set total to find the "only" regions.

You're given information about sets and need to draw the diagram.

> 📋 **Example**
> **In a class of 30 students**:
> - 18 play football
> - 15 play cricket
> - 8 play both
> - Some play neither
>
> Create a Venn diagram and find how many play neither sport.
>
> **Step 1**: Draw two overlapping circles for Football and Cricket inside a rectangle (the universal set of 30 students).
>
> **Step 2**: Place elements:
> - In both (intersection): 8
> - Only in Football: 18 - 8 = 10
> - Only in Cricket: 15 - 8 = 7
> - Neither: 30 - 8 - 10 - 7 = 5
>
> **Step 3**: Answer: 5 students play neither sport.
>
> <JSXGraph id="set-venn-fc" title="Football only=5, Both=3, Cricket only=5" boundingbox={[-6, 4, 6, -4]} axis={false} height={300} code="board.create('polygon',[[-5.5,-3.5],[5.5,-3.5],[5.5,3.5],[-5.5,3.5]],{borders:{strokeColor:fg},fillColor:c.fillGr
> />

Problem Type 2: Using the Union Formula

The formula is efficient when the problem asks for "at least one" or "A or B". If it asks for "neither", find the union first, then subtract from the universal total.

Sometimes it's faster to use [Math: n(A \cup B) = n(A) + n(B) - n(A \cap B)] than to draw a diagram.

📋 **Example**

In a survey of 100 people:

- 60 like chocolate
- 50 like vanilla
- 20 like both
- How many like chocolate OR vanilla (or both)?

Using the formula:

[MathBlock]
n(A \cup B) = 60 + 50 - 20 = 90
[/MathBlock]

Answer: 90 people like chocolate or vanilla.

Problem Type 3: Finding Unknowns

Unknowns in set problems usually represent an overlap or a missing region. Write the formula first, then solve the resulting equation.

Sometimes you know the totals but need to find missing information.

📋 **Example**

In a group of people:

- 100 speak English
- 80 speak French
- [Math: x] speak both languages
- 150 speak English OR French (or both)

Find [Math: x].

Using the formula:

[MathBlock]
n(A \cup B) = n(A) + n(B) - n(A \cap B)
[/MathBlock]

[MathBlock]
150 = 100 + 80 - x
[/MathBlock]

[MathBlock]
150 = 180 - x
[/MathBlock]

[MathBlock]
x = 30
[/MathBlock]

Answer: 30 people speak both languages.

Part 8: Applications to Other Mathematics Topics

Sets appear throughout mathematics. You can use set concepts to solve problems in Number Theory, Algebra, and Geometry.

Sets in Number Theory

Set language is useful in number theory because factors and multiples naturally form groups. Common factors are intersections; combined lists are unions.

📋 **Example**

Find the intersection of sets of factors:

Let [Math: A] = \{factors of 12\} = \{1, 2, 3, 4, 6, 12\}
Let [Math: B] = \{factors of 18\} = \{1, 2, 3, 6, 9, 18\}

[Math: A \cap B] = \{1, 2, 3, 6\} (these are the COMMON factors)

The greatest element in [Math: A \cap B] is 6, which is the GCD(12, 18) = 6

Sets in Number Systems

Number systems are nested sets. This explains why every integer is rational, but not every rational number is an integer.

📋 **Example**

Relationship between number sets:

[Math: \mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}]

- Natural numbers are a subset of integers
- Integers are a subset of rationals
- Rationals are a subset of reals

This shows how number systems nest inside each other!

Sets in Problem Solving

Whenever a question sorts people or objects into categories, a set model can make the wording clearer. Translate the categories before calculating.

Whenever you have categories or groups overlapping, think sets!

📋 Example

Logic problem with sets:

In a school:

- 200 students study Mathematics
- 150 study Science
- 100 study both
- Total students = 300

How many study neither?

Using [Math: n(M \cup S) = 200 + 150 - 100 = 250] study Math or Science.

Students studying neither = 300 - 250 = 50

<JSXGraph id="set-venn-mes" title="Three-set worked Venn (Math/English/Science)" boundingbox={[-6, 6, 6, -5]} axis={false} height={340} code="board.create('polygon',[[-5.5,-4.5],[5.5,-4.5],[5.5,5.5],[-5.5,5.5]],{borders:{strokeColor:fg},fillColor:c.fillGr
/>

📝 Exam Tip

CSEC exam strategy:

- Problem says "both" or "overlap"? Think intersection
- Problem says "either" or "at least one"? Think union
- Problem asks "not in the group"? Think complement
- Use Venn diagrams to visualize before calculating
- For 3+ sets, Venn diagrams are essential

Part 9: Advanced Set Problems

Three Sets with Numbers

Three-set questions require discipline because pair totals usually include the centre. Subtract the "all three" value from each pair before finding the "only" regions.

When working with three sets, keep track of all eight regions carefully.

📋 **Example**

In a group of 100 students:
- 50 study Math
- 40 study English
- 30 study Science
- 15 study Math and English
- 10 study Math and Science
- 8 study English and Science
- 5 study all three subjects

Find:
(a) How many study only Math?
(b) How many study none of these subjects?

Using Venn diagram regions:

Let M = Math, E = English, S = Science

- All three: 5
- Math and English (not Science): 15 - 5 = 10
- Math and Science (not English): 10 - 5 = 5
- English and Science (not Math): 8 - 5 = 3
- Only Math: 50 - 5 - 10 - 5 = 30
- Only English: 40 - 5 - 10 - 3 = 22
- Only Science: 30 - 5 - 5 - 3 = 17
- None: 100 - (30 + 22 + 17 + 5 + 10 + 5 + 3) = 100 - 92 = 8

<JSXGraph id="set-venn-dm" title="De Morgan: (A *B)' = A')B' (shaded outside both)" boundingbox={[-6, 4, 6, -4]} axis={false} height={300} code="board.create('polygon',[[-5.5,-3.5],[5.5,-3.5],[5.5,3.5],[-5.5,3.5]],{borders:{strokeColor:fg},fillColor:c.fillYel
*B)\"],{fontSize:13});" />

Answers:
(a) 30 students study only Math
(b) 8 students study none of these subjects

De Morgan's Laws

De Morgan's laws describe how "not" changes grouped statements. They are especially helpful when a question asks for the complement of a union or intersection.

These important rules show how complements work with unions and intersections:

(A \cup B)' = A' \cap B'

(A \cap B)' = A' \cup B'

In words:

- "Not (A or B)" = "(Not A) and (Not B)"
- "Not (A and B)" = "(Not A) or (Not B)"

📋 **Example**

Let [Math: U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}], [Math: A = \{1, 3, 5, 7, 9\}] (odd), [Math: B = \{3, 6, 9\}]

Verify [Math: (A \cup B)' = A' \cap B']:

Left side: [Math: (A \cup B)']

- [Math: A \cup B = \{1, 3, 5, 6, 7, 9\}]
- [Math: (A \cup B)' = \{2, 4, 8, 10\}]

Right side: [Math: A' \cap B']

- [Math: A' = \{2, 4, 6, 8, 10\}] (evens)
- [Math: B' = \{1, 2, 4, 5, 7, 8, 10\}]
- [Math: A' \cap B' = \{2, 4, 8, 10\}]

Both sides equal \{2, 4, 8, 10\}, confirming the law!