

Vectors

Matthew Williams • Math • May 6, 2026

Vectors describe movement with both size and direction. This makes them useful for journeys, forces, translations, and position on a grid. A vector answer is incomplete if it gives only a length or only a direction.

In the CSEC optional Vectors and Matrices section, vector questions often test whether you can combine movements and interpret the result. Draw arrows when possible, keep direction in mind, and explain whether the vector represents a position from the origin or a displacement from one point to another.

What is a Vector?

A **vector** has two properties:

- 1. **Magnitude** (size/length): How large is it?
- 2. **Direction**: Which way does it point?

Contrast with Scalar: A scalar is just a number (no direction). Examples: temperature, mass, distance.

Vector Notation

Vectors can be represented in multiple ways:

1. Column Matrix (Most Common)

$$\vec{v} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

This means: 3 units right, 4 units up.

2. Row Matrix

$$\vec{v} = (3 \quad 4)$$

3. Component Form

$$\vec{v} = (3, 4)$$

4. i-j Notation (used in physics)

$$\vec{v} = 3\mathbf{i} + 4\mathbf{j}$$

Position and Displacement Vectors

Position Vector of point $P(3, 4)$: The vector from origin to P

$$\vec{OP} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

Displacement Vector from A to B : Tells you how to get from A to B

$$\vec{AB} = \begin{pmatrix} x_B - x_A \\ y_B - y_A \end{pmatrix}$$

Example

From $A(1, 2)$ to $B(5, 6)$:

$$\vec{AB} = \begin{pmatrix} 5 - 1 \\ 6 - 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$$

This means: 4 right, 4 up to go from A to B.

Magnitude of a Vector

Formula: For $\vec{v} = \begin{pmatrix} x \\ y \end{pmatrix}$

$$|\vec{v}| = \sqrt{x^2 + y^2}$$

This is essentially Pythagoras' theorem!

Example

For $\vec{v} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$:

$$|\vec{v}| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

The vector has magnitude 5 (length of 5 units).

Remember

Magnitude is always **non-negative**. It's a measure of size, not direction.

Unit Vectors

A **unit vector** has magnitude 1. It shows pure direction.

To find a unit vector in the direction of \vec{v} :

$$\hat{v} = \frac{\vec{v}}{|\vec{v}|}$$

(Divide the vector by its magnitude)

Example

For $\vec{v} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ with $|\vec{v}| = 5$:

$$\hat{v} = \frac{1}{5} \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 0.6 \\ 0.8 \end{pmatrix}$$

Check: $|\hat{v}| = \sqrt{0.6^2 + 0.8^2} = \sqrt{0.36 + 0.64} = \sqrt{1} = 1$


Vector Operations**Addition and Subtraction**

Addition: Add corresponding components

$$\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a + c \\ b + d \end{pmatrix}$$

Subtraction: Subtract corresponding components

$$\begin{pmatrix} a \\ b \end{pmatrix} - \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a - c \\ b - d \end{pmatrix}$$

 **Example**

$$\begin{pmatrix} 5 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 + 2 \\ 3 + (-1) \end{pmatrix} = \begin{pmatrix} 7 \\ 2 \end{pmatrix}$$

Geometric Meaning: Place vectors head-to-tail; the sum points from start to end.

Scalar Multiplication

Definition: Multiply a vector by a number (scalar)

$$k \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} kx \\ ky \end{pmatrix}$$

Effect:

- If $k > 1$: Vector gets longer (magnified)
- If $0 < k < 1$: Vector gets shorter
- If $k < 0$: Vector reverses direction
- If $k = -1$: Vector points opposite direction (same length)

Example

$$2 \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$$

Original magnitude: 5. New magnitude: $\sqrt{36 + 64} = 10$ (doubled)

$$-\frac{1}{2} \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} -1.5 \\ -2 \end{pmatrix}$$

This vector is half as long and points the opposite direction.

Vectors in Modelling and Problem-Solving

Real-World Application: Navigation

Problem: A plane wants to fly north with velocity 500 km/h, but there's a wind blowing east at 100 km/h. What is the actual velocity?

Solution:

Velocity of plane: $\vec{v}_p = \begin{pmatrix} 0 \\ 500 \end{pmatrix}$ (north)

Velocity of wind: $\vec{v}_w = \begin{pmatrix} 100 \\ 0 \end{pmatrix}$ (east)

Actual velocity: $\vec{v}_a = \vec{v}_p + \vec{v}_w = \begin{pmatrix} 0 \\ 500 \end{pmatrix} + \begin{pmatrix} 100 \\ 0 \end{pmatrix} = \begin{pmatrix} 100 \\ 500 \end{pmatrix}$

Actual direction: Magnitude = $\sqrt{100^2 + 500^2} = \sqrt{260000} \approx 509.9$ km/h

Direction: $\theta = \tan^{-1} \left(\frac{100}{500} \right) \approx 11.3^\circ$ east of north

Real-World Application: Forces

Problem: Two forces act on an object:

- Force 1: $\vec{F}_1 = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ N
- Force 2: $\vec{F}_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ N

Find the resultant force.

Solution:

$$\vec{F}_{resultant} = \vec{F}_1 + \vec{F}_2 = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix} \text{ N}$$

Magnitude: $|\vec{F}| = \sqrt{16 + 36} = \sqrt{52} \approx 7.21 \text{ N}$

Study Vault